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EQUIVALENT-STATIC WIND LOADS FOR LONG-SPAN BRIDGES

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1 INTRODUCTION

Wind loads on long-span bridges generally comprise of mean, direct loads due to wind gusts (referred as background loads) and resonant loads due motion-induced excitations. These dynamic loads at various instances in time tend to stress different bridge parts. They occur simultaneously with other loads such as dead and thermal loads that are essentially acting statically. Therefore, for design purposes, it is convenient to represent all as static loads, decomposed into various equivalent-static loads covering likely critical conditions. Whereas there are various methods [1, 2] for estimation of the motion induced loads, the derivation of the direct loads offers certain theoretical and practical difficulties. This paper presents a technique for derivation of equivalent-static loads including background and motion induced components, based on a time domain simulation of turbulence and buffeting response analysis.

2 BASIC ASSUMPTIONS

For any load combination, structural design can be carried out directly solving

$$[M]\{\ddot{Z}\}+[C]\{\dot{Z}\}+[K]\{Z\}=\{F\}_{\text{Other Loads}}+\{\{F\}_{\text{SE}}+\{F\}_{\text{B}}\}_{\text{Wind}}, \tag{1}$$

where $\{F\}_{\text{SE}}$, represents the self-excited, motion dependant load, $\{F\}_{\text{B}}$, the direct buffeting load, and $[M]$, $[C]$, $[K]$ and $\{Z\}$ have their usual meanings. The self-excited loads include aerodynamic damping and stiffness. The aerodynamic damping may increase or decrease the total damping by a significant margin. This causes problems in the direct integration of Eq. (1) since the aerodynamic damping is frequency dependant and can greatly differ among various vibration modes. Most of the FEA programs today can integrate Eq. (1) directly in time domain but the challenge is converting the modal aerodynamic damping into proportional damping (Rayleigh damping, [3]). Because this difficulty cannot completely be resolved, design based on a direct integration is rarely used and equivalent static loads are often applied instead.

Leaving aside the *Other Loads* and assuming the time histories of structural responses are known, given that any structure will resist external and internal loads with its stiffness alone, Eq. (1) can be rewritten as:

$$[K]\{Z\} = \underbrace{\{F\}_{\mathrm{B}}}_{\text{External}} - \underbrace{[M]\{\ddot{Z}\}}_{\text{Inertia}} - \underbrace{[C + C_{\mathrm{SE}}]\{\dot{Z}\}}_{\text{Damping}}, \tag{2}$$

where the contribution of the aerodynamic stiffness in air is small and can be discarded. Eq. (2) balances in a time varying way where the peak value of the damping term (including aerodynamic and structural damping) does not occur at the same instance with the external and inertial loading. Because structural velocities are out of phase with deflections and accelerations, the damping term can be omitted. The right-hand side of Eq. (2) then simplifies to:

$$\{F\}_{\mathrm{WL}} = \left\{\{\overline{F}\} + \{\tilde{F}\}\right\}_{\mathrm{B}} - [M]\left\{\{\ddot{Z}\}_1 + \{\ddot{Z}\}_2 +,.. + \{\ddot{Z}\}_m\right\}, \tag{3}$$

where $\{\overline{F}\}$ is the mean and $\{\tilde{F}\}$ the gust wind load called *background* and also the *resonant* wind loads of various modes i=1,2..m. Since the peaks in the resonant and background wind loads are statistically independent, Eq. (3) can be transformed to:

$$\{\hat{F}\}_{\mathrm{WL}} = \{\overline{F}\} \pm \sqrt{\{\hat{\tilde{F}}\}^2 + \sum_i \left\{\underbrace{[M]\{\hat{\ddot{Z}}\}_i}_{\{\hat{F}\}_i}\right\}^2}, \tag{4}$$

which provides the peak load envelope using the square root of the sum of the squares (SRSS) method. This formula however is not straightforward for design implementations. The loading envelope is never fully attained in any instance of time but rather it represents the boundary of many possible load distributions. A reasonable technique [2] is then to derive various expected loads covering likely critical loading scenarios k. The equivalent-static load cases are:

$$\{\hat{F}\}_{\mathrm{WL,k}} = \overline{c}\{\overline{F}\} + [\tilde{c}]\{\hat{\tilde{F}}\} + c_{1,k}\{\hat{F}\}_1 + c_{2,k}\{\hat{F}\}_2 + .. + c_{m,k}\{\hat{F}\}_m, \tag{5}$$

where the combination coefficients c are assigned to both background and resonant loads. The mean coefficient $\overline{c}$ is normally set to 1.0 but could be modified due to sheltering effects.

3 COMBINATION COEFFICIENTS OF BACKGROUND LOADS

Wind turbulence or gustiness causes fluctuations in wind loads about a certain mean value. These loads are complex since the gusts are not well correlated along the bridge span. However, via integration of the instantaneous wind loads over the entire bridge structure in a time domain simulation [4], appropriate gust factors gf can be derived:

$$gf_{\mathrm{Load}} = \frac{\overline{F}_{\mathrm{Load}} + pf_{\mathrm{Load}}\sigma_{\mathrm{Load}}}{\overline{F}_{\mathrm{Load}}}, \tag{6}$$

where pf_{Load} is the peak factor and σ_{Load} is the standard deviation (or root-mean-square) of a given load. It should be noted that for any *Load* being Drag, Lift, Torsional Moment, etc., its peak factor will be different. An accurate technique for estimating of these factors is via integration over the bridge or part of it for a given load at every time step, followed by a statistical analysis of the resulting time series of overall loading:

$$F_{\mathrm{Load}}(t) = \int_0^L F_{\mathrm{Load}}(t,s)ds, \tag{7}$$

where L could be either part or whole the deck length, tower height, or sometimes when overall gust loading effects are considered, even the whole bridge. The statistical basis of this partial or overall gust load is found as:

$$\overline{F}_{\mathrm{Load}} = \frac{1}{T}\int_0^T F_{\mathrm{Load}}(t)dt, \quad \sigma_{\mathrm{Load}} = \sqrt{\frac{1}{T}\int_0^T \left(\overline{F}_{\mathrm{Load}} - F_{\mathrm{Load}}(t)\right)^2 dt}, \text{ and } pf_{\mathrm{Load}} = E\left[99\%, \left(F_{\mathrm{Load}} - \overline{F}_{\mathrm{Load}}\right)\right] \tag{8}$$

The peak factor is calculated from the probability distribution curve E with a 99% probability. These peak factors are then applied on the mean wind loads to account for the direct gust loading on the bridge. The background forces and moments are derived multiplying the corresponding mean loads by:

$$gf_{\mathrm{Load,BG}} = \left(gf_{\mathrm{Load}} - 1\right), \text{ to obtain } \tilde{F}_{\mathrm{Load}} = gf_{\mathrm{Load,BG}} \bar{F}_{\mathrm{Load}}. \quad (9)$$

4 COMBINATION COEFFICIENTS FOR INERTIAL LOADING

Reference [2] suggests $c_{j,k} = \pm 1.0$ for only one modal term, ±0.8, for two terms, ±0.6 for four and more terms. There values are close to the expansion $\frac{\sqrt{m}}{m}$, where m is the number of modal terms including background loads. Alternatively the $\frac{\sqrt{m-1}}{m-1}$ formula has been also proposed. Table 1 presents rounded values provided by these formulae, which bound the range typically found during aeroelastic model tests of long span bridges.

m	$\frac{\sqrt{m}}{m}$	$\frac{\sqrt{m-1}}{m-1}$
1	1.0	--
2	0.70	0.45
3	0.60	0.40
4	0.50	0.35
5	0.45	0.30

Table 1: Load combination factors

5 LOAD COMBINATIONS

Various combinations can be developed with the load patterns on the bridge being distributed lateral, vertical, longitudinal, and torsional loads. Each of these loads can represent an individual worst case in terms of the lateral or vertical loading on the deck, lateral loading on the towers, or torsion, with various combinations of the bridge modes of vibration. The load patterns include symmetric and asymmetric loads over various parts of the bridge. For design, these loads are applied simultaneously as static loads in combination with other structural loads, and thereafter each main structural member is designed based on the worst loading combination (i.e., stress and strain). Based on our experience with aeroelastic model tests, any load pattern should include:

- the mean wind load; plus
- one principal dynamic mode of full value and 1 to 3 subordinating modes with combination coefficients in the range of values as shown in Table 1.

When composing load combinations other rules will also apply such as:

- modes with similar shapes are only combined which allows for minimal combinations;
- the loading envelope according to Eq. (4) should not be overly exceeded, therefore values lower than the highest values in Table 1 can be applied;
- to reduce the number of load cases in some instances, coefficients higher by about 10% can be applied to cover difficult "corners" of the loading envelope; and
- a sufficient number of combinations should be assembled to cover all branches of the loading envelope (not simultaneously).

Load Case	*c* - Mean Load	$\tilde{c}$ - Background			Combination factor *c* for various modes & shapes							
		L	*V*	*T*	1	2	3	4	5	6	7	8
					L1	V1	L2	V2	T1	L3	V3	T2
1	1.0	0.5	0.5	0.5	1.0	0.5	0.0	0.0	0.5	0.0	0.0	0.0
2	1.0	0.5	0.5	0.5	0.5	1.0	0.0	0.0	0.5	0.0	0.0	0.0
3	1.0	0.5	-0.5	0.5	0.5	-1.0	0.0	0.0	0.5	0.0	0.0	0.0
4	1.0	0.5	0.5	0.5	0.5	0.5	0.0	0.0	1.0	0.0	0.0	0.0
5	1.0	0.5	0.5	-0.5	0.5	0.5	0.0	0.0	-1.0	0.0	0.0	0.0
6	1.0	1.0	1.0	1.0	0.5	0.5	0.0	0.0	0.5	0.0	0.0	0.0
7	1.0	0.5	0.5	0.5	0.0	0.0	1.0	0.5	0.0	0.0	0.0	0.5
8	1.0	0.5	0.5	0.5	0.0	0.0	0.5	1.0	0.0	0.0	0.0	0.5
9	1.0	0.5	0.5	0.5	0.0	0.0	0.5	-1.0	0.0	0.0	0.0	0.5
10	1.0	0.5	0.5	0.5	0.0	0.0	0.5	0.5	0.0	0.0	0.0	1.0
11	1.0	0.5	0.5	0.5	0.0	0.0	0.5	0.5	0.0	0.0	0.0	-1.0
12	1.0	1.0	1.0	1.0	0.0	0.0	0.5	0.5	0.0	0.0	0.0	0.5
13	1.0	0.6	0.6	0.6	0.0	0.0	0.0	0.0	0.0	1.0	0.6	0.0
14	1.0	0.6	0.6	0.6	0.0	0.0	0.0	0.0	0.0	0.6	1.0	0.0
15	1.0	1.0	1.0	1.0	0.0	0.0	0.0	0.0	0.0	0.6	0.6	0.0

Table 2: Example of load combinations (single span bridge considered, the list of combinations is not complete)

In Table 2, L/V/T denote Lateral/Vertical/Torsional mode, and the number shows its order of appearance, not its shape, for example V2 is the 2nd mode with predominant vertical motions.

6 CONCLUSIONS

The method of equivalent-static loads is formulated starting from the fundamental equation of equilibrium, being extended to account fully for the background loads. It must be noted that loads due to drag, lift and moment should be represented by different gust factors that are difficult to obtain using conventional methods. The presented method will typically produce about 10 to 30 load cases depending on the complexity of given bridge. It should be noted that on many bridges (especially during construction) the modes are highly coupled and their separation into branches of lateral, vertical, and torsional modes is often difficult. Nevertheless the combination technique described above is fully applicable with a caution when reducing the number of combinations based on the structural symmetry.

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