EXPERIMENTAL AND COMPUTATIONAL ENHANCEMENT FOR HYBRID ANALYSIS OF SUSPENSION BRIDGES

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Keywords: Aeroelasticity, Long-span Bridges, Flutter, Sectional Tests.

1 INTRODUCTION

The hybrid methods used in the aeroelastic analysis of flutter for long-span bridges are computational, but they use coefficients and functions obtained experimentally in a wind tunnel. The experimental testings are carried out with a deck sectional model and there are two types: an aerodynamic testing for obtaining the aerodynamic coefficients such as drag, lift and moment, and an aeroelastic testing for obtaining the flutter derivatives by studding the model oscillations under wind action. In this article, some improvements were achieved in the experimental phase as well as in the computational phase of the method. The influence of variation of the aerodynamic coefficients were studied with Reynold’s number; different sets of springs were used to include a wide range of reduced velocities that the flutter derivatives depend on; the influence of deformation that the static wind load produces at angle of attack along the bridge was studied; finally the variation of the angle of attack was taken into account to determine the critical flutter speed. These improvements were applied to the sectional and computational models of the future Messina Strait Bridge.

2 AERODYNAMIC SECTIONAL TESTING

The aerodynamic coefficients are obtained by carrying out a testing of a fixed deck sectional model measuring inside the wind tunnel drag force $D_s$, lift $L_s$ and moment $M_s$ that exerts air flow over the model. A scale bridge deck sectional model is built, whose shape should be as similar to the prototype as possible. Figure 1 shows a sectional model of the Messina Bridge. The mass does not need to be scaled to the original bridge since the displacements are constrained and there are no inertial forces. We simply search for simulating the real boundary conditions that determine the air flow around the deck. It is essential to determine the correct air flow velocity in the wind tunnel to obtain the aerodynamic forces. This should be carried out by trying with different angles of attack as shown in figure 2 for the Messina Strait Bridge. In the graph, it can be observed that with Reynold’s number over 400000, the values do not vary noticeably. Once the velocity is determined, 11m/s for the Messina example
which corresponds to \( Re = 460000 \), the testings were carried out varying the angle of attack to obtain the aerodynamic coefficient graphs like the one shown in figure 2.

![Figure 1: Sectional deck model of the Messina Strait Bridge.](image)

**Figure 2: Left) Variation of the aerodynamic coefficients for different angles of attack in function of Reynold's number. Right) Aerodynamic coefficients for \( Re =46000 \).**

### 3 AEROELASTIC SECTIONAL TESTING

The aeroelastic sectional testings under free vibration consist of sustaining the sectional model by springs and make them freely oscillate with and without air flow inside the wind tunnel. A detailed explanation of this procedure is found in Jurado, Leon and Hernandez [1]. From the displacements of the model, we can calculate the stiffness properties and damping, for example with the MITD (Modified Ibrahim Time Domain Method), and from the variation of those properties as the wind velocity varies, we can obtain the flutter functions.

A sectional model requires less similarity conditions than a reduced complete bridge model. The geometric similarity is essential and the model length should be three times its width in order to be considered two-dimensional. There is no need to consider a scale of masses since we only try to quantify the wind action in function of the oscillatory movements of the deck. The model can be elastically sustained using eight to twelve springs: four or eight vertical and four horizontal ones.

The stiffness of the springs determines the vibration frequencies \( 2\pi f = \omega \) of the system that together with the wind velocity in the tunnel \( V \) and the model width \( B \), determines the range of reduced velocities \( V* = V/B = 2\pi V/\omega B = 2\pi K \) for the flutter functions. For example, eighteen flutter functions of the deck model of the Messina Bridge were obtained for angles of attack of the equilibrium position: \(-3^\circ, 0^\circ\) and \(+3^\circ\). Four types of supporting were used in order to vary the natural frequencies and get a wide range of reduced velocities. The testings were carried out for wind velocities between 6 and 20 m/s.

Natural frequencies of the model and limits of the reduced velocity for flutter functions are shown in Table 1 for each test. With the tests 2 and 3 of three degrees of freedom, 18 flutter functions are obtained simultaneously, while with the test 4 of only rotational freedom, we
can only identify the flutter functions, $A2^*$ and $A3^*$. Figure 3 shows some obtained flutter functions.

\[
\begin{array}{|c|c|c|c|c|}
\hline
(A_5^*, A_8^*, H_5^*, H_6^*, P_1^*, P_4^*) & (A_1^*, A_4^*, H_1^*, H_4^*, P_5^*, P_6^*) & (A_2^*, A_3^*, H_2^*, H_3^*, P_2^*, P_3^*) \\
\hline
f_v, f_w, f_\phi, V^*(f_v)_{\text{min}}, V^*(f_v)_{\text{max}}, V^*(f_w)_{\text{min}}, V^*(f_w)_{\text{max}}, V^*(f_\phi)_{\text{min}}, V^*(f_\phi)_{\text{max}} \\
\hline
\text{Test 1} & 2.2 & 6.8 & 4.521 & 15.07 & 3.40 & 11.34 & 1.45 & 4.85 \\
\text{Test 2} & 2.8 & 1.7 & 3.54 & 11.79 & 5.82 & 19.41 & 2.82 & 9.39 \\
\text{Test 3} & 1.3 & 1.4 & 7.62 & 25.38 & 7.22 & 24.09 & 4.90 & 16.34 \\
\text{Test 4} & 0 & 0 & 1.14 & - & - & - & 8.68 & 28.95 \\
\hline
\end{array}
\]

Table 1: Natural frequencies in each testing and range of reduced velocities for the flutter functions.

4 INFLUENCE OF THE STATIC DEFORMATION ON THE ANGLE OF ATTACK

Rotation about the axis of the deck can vary some degrees along the deck. For example, $0.6^\circ$ for the Messina Bridge [2] at the centre span. The deck rotation causes a change in the angles of attack, which affects the aerodynamic coefficient and the flutter coefficient values. The calculation of the static solution of the wind action is carried out by solving the system expressed by:

\[
K(u)u = f[a(u)]
\]

where not only the stiffness matrix, $K$ depends on the displacements $u$, but also the wind loads $f$ that vary with the angle of attack $\alpha$ along the deck and therefore also depend on $u$. An approximation of the problem (1) consists of considering the deck deformation only in the wind forces.

\[
Ku = f[a(u)]
\]

In this approximation, the influence of the wind forces on the structural stiffness is not considered. The problem is resolved by iterations. This method was applied to the static deformation of the Messina Bridge observing noticeable differences in the deck displacements for the wind velocity over 50m/s as shown in Figure 4.
5 FLUTTER ANALYSIS

As it was explained in Jurado and Hernández [3], the flutter condition is obtained by the computational solving of a non linear eigen-problem which comes from the dynamic equilibrium equation for the deck. The imaginary part of an eigenvalue takes into account the frequency, while its real part is associated with the damping ratio. The condition of flutter corresponds to the lowest wind speed \( U_f \) which gives one eigenvalue with vanished real part. An improvement of the flutter analysis is to consider the dependency of the flutter derivatives on the angle of attack at each point of the deck due to the static deformation. Figure 5 shows graphs with the evolution of the eigen-values for increasing wind velocities with or without varying the angle of attack. For Messina Bridge the critical flutter velocity increases favorably when this effect is taken into account.

![Figure 5: Relation between the real and imaginary part of the eigenvalues in the flutter analysis not considering \( U_f = 100 \text{m/s} \) and considering \( U_f > 120 \text{m/s} \) the variation of angle of attack.](image)

6 CONCLUSIONS

- The wind velocity used for the aerodynamic sectional testing should be chosen by previously studying the variation of aerodynamic forces in function of Reynold’s number.
- Various sets of springs in the aeroelastic sectional testings permit a bigger range of reduced velocities in obtaining the flutter derivatives.
- The static deformation due to the wind load affects considerably the angle of attack at high wind velocities close to the flutter.
- The critical flutter velocity also varies noticeably if the variation of angle of attack along the bridge span is taken into account.

ACKNOWLEDGEMENTS

This research has been funded by the Galician Council of Innovation, Trade and Industry under project PGIDIT04CCP118002PR.

REFERENCES

