New Consideration on Flutter Properties basing on SBS -Fundamental Flutter Mode, Similar Selberg's Formula, Torsional Divergence Instability, and New Coupled Flutter Phenomena affected by Structural Coupling

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1 INTRODUCTION

In this paper, by using SBSA[1], physical meanings of coupled flutter branch are discussed in relation to fundamental flutter modes. And, it is clarified that torsional divergence is classified into static 1DOF divergence and dynamic 2DOF divergence. Moreover, similar form of Selberg's formula [2] can be obtained basing on Torsional Branch (TB) characteristics. Focusing on the structural properties of horizontal and torsional displacement, 2DOF flutter instability affected by structural coupling have been considered in stead of 3DOF analysis.

2 FUNDAMENTAL FLUTTER MODE

Torsional fundamental mode is defined as substantially torsional vibration around certain point apart from mid-chord point. In this mode, the phase difference, between torsional and heaving response at mid-chord point, is 0° or 180° and torsional twist center is upstream point or downward point from the mid-chord point, respectively. These fundamental modes are expressed by T_0 or T_{180} as shown in Fig.1(a), (b). On the other hand, heaving fundamental mode is defined as prominent heaving response induced by lift generated by slight pitching angle in quasi-steady sense with -90° or 90° as the phase difference as shown in Fig.1(c), (d). By taking into account of fundamental mode definition, the flutter mode in coupled flutter can be resolved into two fundamental modes by using phase difference as eq.1.

$$\phi = \phi_0 \sin \omega t,\ \eta = \eta_0 \sin(\omega t - \Psi) \tag{1}$$

$$\begin{aligned} &T_{180} = -\cos\Psi,\ H_{90} = \sin\Psi\ (90^{\circ} \le \Psi \le 180^{\circ}) \quad T_0 = \cos\Psi,\ H_{90} = \sin\Psi\ (0^{\circ} \le \Psi \le 90^{\circ}) \\ &T_0 = \cos\Psi,\ H_{-90} = -\sin\Psi\ (-90^{\circ} \le \Psi \le 0^{\circ}) \quad T_{180} = -\cos\Psi,\ H_{-90} = -\sin\Psi\ (-180^{\circ} \le \Psi \le -90^{\circ}) \end{aligned} \tag{2}$$

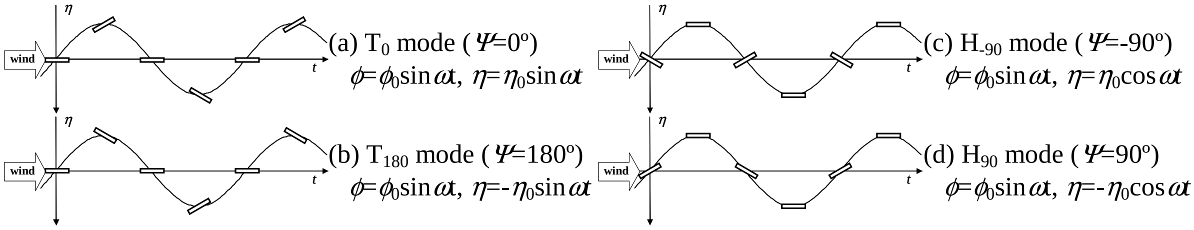


Fig.1 Fundamental flutter modes in torsional and heaving 2DOF coupled flutter

3 SIMILAR FORMULA WITH SELBERG'S FORMULA TO PREDICT THE FLUTTER ONSET VELOCITY OF THIN PLATE

The following lines show discussion about Selberg's formula[2] which is a well-known evaluation formula of flutter onset velocity for thin plate, as described eq.3.

$$V_{cr} = 3.71 f_{\phi 0}(2b)\sqrt{(\sqrt{mI}/\rho(2b)^3)(1-(f_{\eta 0}/f_{\phi 0})^2)} \tag{3}$$

Then, authors investigate the formula to predict the flutter onset velocity by using SBSA. It is clarified that flutter onset velocity is almost agreed to crossing point of V-ω_η diagram and V-ω_ϕ diagram of TB, as shown in Fig.2. Then, the following assumptions are used.
Assumption1: Quasi-steady state; $F(k)$=1, and $G(k)$=0, where $C(k)= F(k)-iG(k)$.
Assumption2: Flutter occurs, when crossing point of ω_η-V diagram and ω_ϕ-V diagram.
Assumption3: $\omega_\eta=\omega_{\eta0}$ at any velocity and ω_ϕ in 2DOF can be expressed by ω_ϕ in 1DOF.

Using Assumption1, torsional 1DOF frequency is described as eq.4.

$$\omega_\phi=\sqrt{\omega_{\phi0}^2/(1+(\rho b^4/I)A_3^*)}=\sqrt{\omega_{\phi0}^2/(1+(\rho b^4/I)(\pi/k^2))} \quad (4)$$

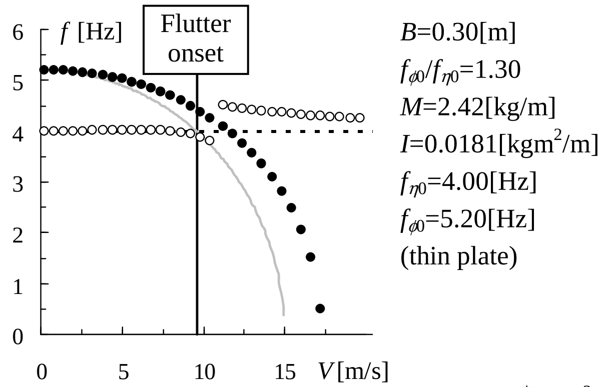
When flutter occurs, using Assumption2 and Assumption3, the following equation can be derived. Where, $k=b\omega_F/V=b\omega_\eta/V=b\omega_{\eta0}/V$ (because of TB).

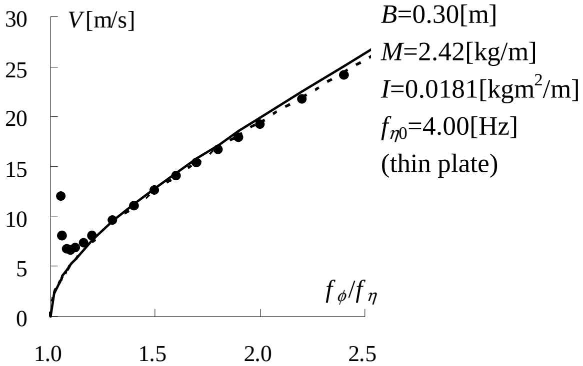
$$V_{cr}=f_{\phi0}\sqrt{(4\pi I)(1-(f_{\eta0}/f_{\phi0})^2)/(\rho b^4)} \quad (5)$$

Relationships between m and I in thin plate are described as $I=mb^2/3$, and eq.5 can be modified like Selberg's Form as follows;

$$V_{cr}=3.81 f_{\phi0}(2b)\sqrt{(\sqrt{mI}/\rho(2b)^3)(1-(f_{\eta0}/f_{\phi0})^2)} \quad (6)$$

As shown in Fig.3, the flutter onset velocity of thin plate obtained by flutter analysis, Selberg's formula, and eq.6. From this figures, flutter onset velocity obtained by eq.6 fairly well agrees to that obtained by SBSA as well as Selberg's formula.





●: torsional freq. in HB —: torsional freq. ($A_3^*=\pi/k^2$) ○: flutter freq. f_F in HB ·····: heaving natural freq. $f_{\eta0}$

Fig.2: Frequency characteristics in TB

●: flutter onset velocity —: Equation (6) ·····: Selberg's formula

Fig.3: Comparison with flutter onset velocity

4 TORSIONAL DIVERGENCE

Divergence could occur when restoring force become smaller than moment force in static state. For dynamic system, when the torsional rigidity becomes zero due to aerodynamic unsteady moment, divergence occurs. Therefore, by using SBSA, static divergence and dynamic divergence can be studied. Divergences are classified into following three different types.

(a) 1DOF Divergence: Torsional rigidity is zero at Step1 of TB.

$$\omega_\phi^2=\omega_{\phi0}^2-(\rho b^4/I)\omega_\phi^2 A_3^*=0 \quad (7)$$

(b) 2DOF dynamic divergence: Torsional rigidity is zero at Step3 of TB.

$$\omega_F^2=\omega_{\phi0}^2-(\rho b^3/I)\omega_F^2\{bA_3^*-A_4^*(\eta_0/\phi_0)(\cos\Psi-\zeta_F\sin\Psi)-A_1^*(\eta_0/\phi_0)(\zeta_F^2+1)\sin\Psi\}=0 \quad (8)$$

(c) 1DOF dynamic divergence: Torsional rigidity is zero at step2 of HB.

$$\omega_\phi^{*2}=\omega_{\phi0}^2-(\rho b^4/I)\omega_F^2 A_3^*=0 \quad (9)$$

5 2DOF COUPLED FLUTTER AFFECTED BY STRUCTURAL COUPLING OF FULL EALSTIC SUSPENSION BRIDGE WITH TRUSS-STIFFENED GIRDER

Recently coupled flutter instability has been analyzed in 3DOF that is vertical displacement η, horizontal displacement ξ, and torsional displacement ϕ, instead of conventional 2DOF (η, ϕ) [3]. The background of 3DOF analysis stands on the experimental results on flutter characteristics of Akashi Strait Bridge elastic full-scale model (AFM, 1/100 scale, 40m total span length) as shown in Photo 1. It has been reported that damping-velocity characteristics could not always be explained by conventional 2DOF analysis [3]. 3DOF analysis taking into account of horizontal motion and aerodynamic forces caused by horizontal motion of bridge girder, can show better fitting to test results as shown in Fig.4 [4]. Furthermore, looking the video film of flutter characteristics of AFM, flutter mode is significantly similar to those in 2DOF system of flat rectangular cylinder. In consequence, flutter of AFM could be conventional 2DOF coupled flutter, and the discrepancy of flutter

Photo1: Akashi Full-scale Model, 1/100 scale, 40m total span length (courtesy of Honshu-Shikoku BEC)

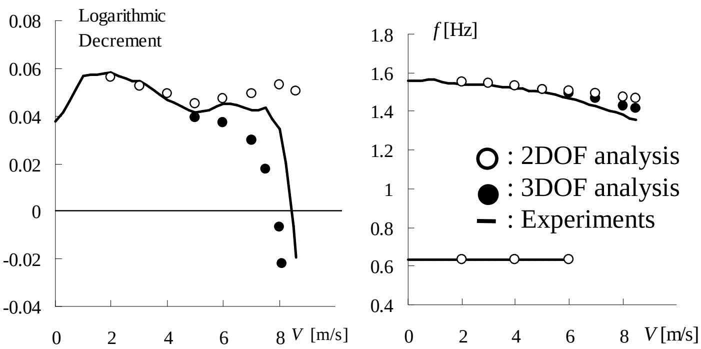
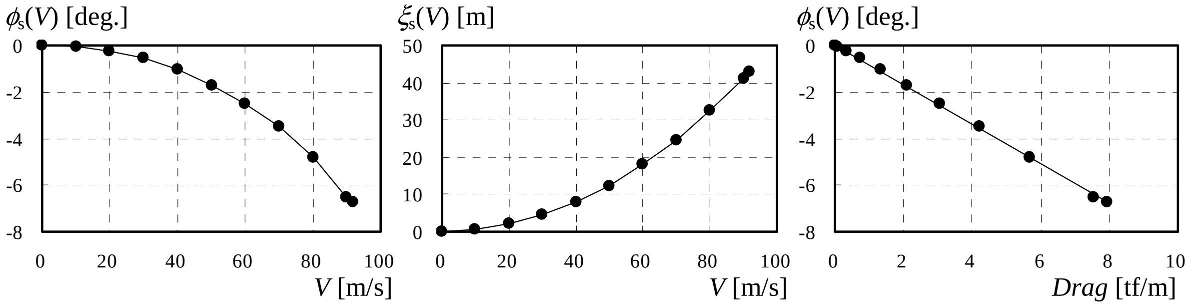


Fig.4: Results of analyses and experiments at AFM (Normal girder) [2]

onset velocity between test results and analytical results could be caused by another reason. Therefore, in this study, focusing on the horizontal and torsional displacement, 2DOF aerodynamic coupling and structural coupling have been considered in flutter analysis.

Looking at static displacement characteristics as shown in Fig.5 (a), (b), when wind velocity increases, static horizontal displacement $\xi_s(V)$ increases. And static torsional displacement $\phi_s(V)$ is induced by this horizontal displacement $\xi_s(V)$. This horizontal displacement $\xi_s(V)$ is caused by drag force. Therefore, static torsional displacement $\phi_s(V)$ due to horizontal displacement $\xi_s(V)$ can be thought to be linear relation to drag force as shown in Fig.5(c).



(a): Torsional displacement $\phi_s(V)$ (b): Horizontal displacement $\xi_s(V)$ (c): $Drag$-$\phi_s(V)$ relation

Fig.5 Static displacement characteristics at middle point of AFM (angle of attack = 0 [deg.])

This $Drag$-$\phi_s(V)$ characteristics should be substantially structural feature as structural coupling between horizontal displacement ξ, and torsional displacement ϕ of this AFM. By the influence of structural coupling characteristics, additional torsional displacement ϕ^* is generated by the change of drag force. While torsional response ϕ changes the drag force from quasi-steady base, therefore, the additional moment term $M^*(\phi^*)$ induced by additional torsional displacement ϕ^* should be added in torsional differential equation as follows;

$$\begin{aligned} m\ddot{\eta}+C_\eta\dot{\eta}+k_\eta\eta &= \tfrac{1}{2}\rho V^2(2b)[kH_1^*\tfrac{\dot{\eta}}{V}+kH_2^*\tfrac{b\dot{\phi}}{V}+k^2H_3^*\phi+k^2H_4^*\tfrac{\eta}{b}] \\ I\ddot{\phi}+C_\phi\dot{\phi}+k_\phi\phi &= \tfrac{1}{2}\rho V^2(2b^2)[kA_1^*\tfrac{\dot{\eta}}{V}+kA_2^*\tfrac{b\dot{\phi}}{V}+k^2A_3^*\phi+k^2A_4^*\tfrac{\eta}{b}]+M^*(\phi^*) \end{aligned} \quad (10)$$

Taking into account of the relationship diagram between drag force *Drag* and torsional displacement $\phi_S(V)$, additional torsional displacement ϕ^* caused by structural coupling is calculated by the following.

$$\phi^*=-0.0148\times Drag(\phi) \quad (11)$$

Where coefficient (-0.0148) is obtained from structural property as shown in Fig.5 (c). Finally, additional moment term $M^*(\phi^*)$ is considered as the elastic force.

$$M^*(\phi^*)=k_\phi\phi^*=-0.0074\times\rho V^2 D\tfrac{dC_D}{d\alpha}\omega_{\phi 0}^2\phi \quad (12)$$

As shown in eq.10 and 12, additional torsional displacement ϕ^* possess the effect to decrease torsional rigidity. Then, 2DOF SBSA is conducted by using the aerodynamic derivatives of Akashi Strait Bridge girder at angle of attack 0 deg. The results on velocity-damping (V-δ) diagram and velocity-frequency (V-ω) diagram obtained by eq.10 are compared with test results [2] in Fig.6. Results show good agreement, in particular, at near and after flutter onset, rapid decreasing characteristics of damping as velocity increases, can be well calculated by using eq.10. In conclusion, the author would like to emphasize that as far as the coupled flutter of AFM should not be 3DOF coupled flutter from aerodynamic point of views, but aerodynamically 2DOF coupled flutter strongly affected by structural coupling feature between horizontal and torsional response.

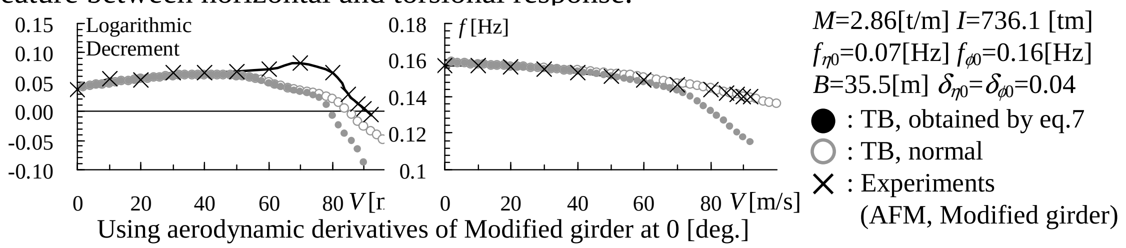


Fig.6: results of 2DOF (1st symmetric torsional and heaving modes) SBSA with structural coupling

6 CONCLUSION

Using fundamental flutter modes, any vibration can be resolved into torsional mode T_0 or T_{180} and heaving mode H_{-90} or H_{90}. As far as thin plate, from the point of view of self excited term, T_0 mode and H_{-90} mode play major roles in TB and HB. Basing on HB characteristics, similar form of Selberg's formula can be provided. And, three different torsional divergence velocities are shown by SBSA. The coupled flutter of AFM can be characterized by aerodynamically conventional 2DOF coupling and structural coupling. Simplified analytical model developed in this study fairly well explains flutter characteristic obtained by wind tunnel tests.

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