BBAA VI International Colloquium on:
Bluff Bodies Aerodynamics & Applications
Milano, Italy, July, 20-24 2008

RECENT INVESTIGATIONS ON LONG-SPAN BRIDGE AEROELASTICITY IN THE PRESENCE OF TURBULENCE FIELDS WITH UNCERTAIN SPAN-WISE CORRELATION

Luca Caracoglia

Department of Civil and Environmental Engineering
Northeastern University, Boston, Massachusetts, 02115 USA
e-mail: lucac@coe.neu.edu

Keywords: Long-Span Bridges; Aeroelastic Response; Span-Wise Turbulence Correlation; Stochastic Calculus.

Abstract: *The dynamic response of a long-span bridge under the effects of a turbulent wind field with uncertain span-wise correlation is investigated. The interaction between two structural modes only, a purely flexural and a purely torsional one, is preliminarily considered. The steady-state dynamic response solution is derived by simulating the structural response under wind turbulence as an equivalent Markov-type multivariate process. This allows for the representation of the turbulence-field characteristics corresponding to an "unconventional storm", which may differ from the scenario employed during bridge design.*

1 INTRODUCTION

Fluid-structure interaction problems related to long-span bridges are governed by multi-physics equations involving the modeling of the wind turbulence flow, the motion-induced-loading mechanism and the structural response. Among these problems the response due to turbulence-induced wind pressures (i.e., buffeting) is of particular relevance since it involves the serviceability limit states of the structure. Analytical and numerical multimode techniques are readily available and have been successfully applied for the solution of this problem under the assumptions of linear response and linearized fluid-dynamic interaction (e.g., [1]).

Nevertheless, recent comparisons between buffeting response simulations and full-scale vibration records and have highlighted some discrepancies (e.g., [2,3]). These can be related to the fact that the parameters for the description of the approaching wind field, such as the turbulence power spectral density and the aerodynamic coefficients of the deck cross-section, are affected by uncertainty. Sources of uncertainty are linked to either an incomplete description of the phenomenon or the extraction of the input parameters through experiments.

A recent research initiative focuses on the development of a methodology capable of directly incorporating the uncertainty associated with these input parameters into the equations[4]. As part of this investigation, the direct derivation of the joint probability density of the response variables has been proposed as a particularly suitable approach, and as an alternative to numerical Monte Carlo simulations. The proposed method is capable of estimating the statistical moments of the response through truncation (closure), when the information on the statistical distribution of the input random parameters is not fully available. While this method has been used for the study bridge flutter (e.g., [5]), little attention has been devoted to buffeting.

In this study the dynamic response of a long-span bridge under the effects of a turbulent wind field with uncertain span-wise correlation is investigated. The interaction between two modes only, namely a purely flexural and a purely torsional one is considered to investigate the effects of modal coupling and to describe response of a long-span bridge in the proximity of coupled-flutter instability. A steady-state dynamic response solution is derived by simulating the structural response under wind turbulence as an equivalent Markov-type multi-variate process. The derivation of the corresponding Reduced Fokker-Plank Equation (RFPE) via stochastic calculus [6] is discussed, in which random processes and variables include modal response and uncertain span-wise spatial correlation of the turbulence.

2 PROBLEM FORMULATION

The dynamic response of a long-span bridge was simulated in the time domain. The representation of aeroelastic forces (self-excited drag D_{AE}, lift L_{AE} and moment M_{AE} per unit length), is conventionally defined in a mixed time-frequency formulation as a linear combination of p lateral, h vertical and α angular displacement components and flutter derivatives P_q*,H_q*,A_q* (q=1,…,6) as shown in Fig. (1). As an example, L_{AE} is (e.g., [1])

$$L_{AE}=0.5\rho U^2 B\left[KH_1^*\dot{h}/U+KH_2^*B\dot{\alpha}/U+K^2H_3^*\alpha+K^2H_4^*h/B+KH_5^*\dot{p}/U+K^2H_6^*p/B\right]. \quad (1)$$

In Eq. (1) ρ denotes the density of the air, U is the cross-flow reference mean wind velocity, B is the deck width and t is the time. The derivatives are all functions of the reduced frequency $K=\omega B/U$ with ω circular frequency. Similarly turbulence-induced forces are represented in terms of quasi-steady theory. The loading depends on the vertical (w) and lateral (u) turbulence components and is obtained by linear expansion about the equilibrium position under static wind, such as for the lift buffeting force L_b,

$$L_b=0.5\rho U^2 B\left[2C_L\,u(x,t)/U+\left(\hat{C}_L+C_D\right)w(x,t)/U\right], \quad (2)$$

with x longitudinal bridge axis coordinate, C_L and C_D drag and lift static coefficients per unit length at initial angle of attack $\alpha_0=0$, and $\hat{C}_L$ =dC_L/dα (computed at α_0).

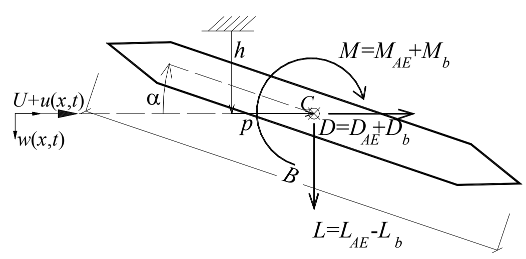

Figure 1: Aeroelastic and aerodynamic actions on a bridge deck.

Aeroelastic forces are converted into dimensionless time domain ($s=tU/B$) quantities via indicial-function formulation. As an example the indicial lift due to vertical-velocity unit step variation at s=0 is $\Phi_{Lh}(s)=c_{0,Lh}-\sum_i c_{i,Lh}\exp(-d_{i,Lh}s)$. The parameters $c_{i,Lh}$ and $d_{i,Lh}$ are derived from non-linear regression of the corresponding derivatives. The dynamic motion components of the deck (Fig. 1) are represented via superposition of still-air structural eigen-functions, $h_g(x)$, $p_g(x)$ and generalized coordinates $\xi_g(t)$, as

$$p = \sum \xi_g(t) B p_g(x); \quad h = \sum \xi_g(t) B h_g(x); \quad \alpha = \sum \xi_g(t) \alpha_g(x). \tag{3}$$

The response of the bridge is analyzed by considering the interaction between two modes only, namely a purely flexural and a purely torsional one, thereby allowing the preliminary investigation of the effects of modal coupling. The two dominating modes are denoted as j and k with $h_j(x) \neq 0$ and $\alpha_k(x) \neq 0$ exclusively, whereas other eigen-components (Eq. 3) are identically set to zero.

Turbulence-induced loading (Eq. 2) was recast into corresponding generalized forces (modes j and k) as a function of $\hat{u}(s) = u(s)/U$ and $\hat{w}(s) = w(s)/U$ and span-wise correlation terms depending on the selected mode. As an example, the generalized force correlation term associated with mode j and corresponding to L_b is defined (in a least-square sense), as

$$\left[(\hat{C}_L + C_D) L_{\psi_{hj}}\right]^2 = \iint (\hat{C}_L + C_D)^2 h_j(x) h_j(\sigma) \exp(-\tilde{c}\,|x-\sigma|/l)\, d\sigma\, dx / l^2 . \tag{4}$$

In Eq. (4) the summation is performed over the entire bridge span length l, and $\tilde{c} \approx 5\omega_j l/(2\pi U)$ with ω_j structural frequency of mode j. Direct influence of lateral (drag) forces and chord-wise aerodynamic admittance are neglected in this study.

3 STEADY-STATE PROBABILITY DENSITY FUNCTION VIA RFPE

The span-wise correlation of aerodynamic forces for mode j, $L_{\psi hj}$, is assumed as a random variable to simulate the effects of an uncertain wind scenario. The equation of motion can be recast into a first-order Itô-type stochastic differential system through variables $\mathbf{Z}(s) = \left[\mathbf{Z}_{AE}(s), \mathbf{Z}_{TB}(s)\right]^T$, with $\mathbf{Z}_{AE}(s) = \left[\xi_j, \xi_k, \xi'_j, \xi'_k, \nu_{ae,g,pq,i}, \mu_{ae,g,pq,i}\right]^T$ the modal and aerolastic terms, while $\mathbf{Z}_{TB}(s) = \left[L_{\psi_j}\hat{u}(s), L_{\psi_j}\hat{w}(s), \hat{u}(s), \hat{w}(s), L_{\psi_j}\right]^T$ are the turbulence-dependent quantities. The prime symbol denotes the derivative with respect to s and $[\bullet]^T$ is the transpose operator. State augmentation is used [6].

The quantities $\nu_{ae,g,pq,i}(s)$ and $\mu_{ae,g,pq,i}(s)$ are vertical-velocity and torsional-angle aeroelastic states [5], respectively, with g=j,k, p,q=h,α and i=1..M_{pq} number of exponential time lags, which can be rewritten in state space for the first process as $\nu'_{ae,pq,g,i} = d_{i,pq} c_{i,pq} \xi'_g - d_{i,pq} \nu_{ae,pq,g,i}$. Chord-wise admittance is not addressed at this stage but can be readily included via additional aerodynamic states. A n-th dimensional linear stochastic differential system of equations [6] is derived, in which the input is represented as a univariate Wiener process $W(s)$,

$$d\mathbf{Z}(s) = \mathbf{A}\mathbf{Z}(s)ds + \sqrt{2\pi}\left[\mathbf{C}\mathbf{Z}(s) + \mathbf{d}\right] dW(s), \tag{5}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A_{AE}}^{\mathrm{r\times r}} & \mathbf{A_{TB}}^{\mathrm{r\times(n-r)}} \\ \mathbf{0}^{\mathrm{(n-r)\times r}} & \mathbf{T}^{\mathrm{(n-r)\times(n-r)}} \end{bmatrix}, \mathbf{T}^{\mathrm{(n-r)\times(n-r)}} = \begin{bmatrix} \mathbf{T}_{uw} & \mathbf{0}^{2\times 2} & 0 \\ \mathbf{0}^{2\times 2} & \mathbf{T}_{uw} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \mathbf{T}_{uw} = \begin{bmatrix} -G_{1u} & 0 \\ 0 & -G_{1w} \end{bmatrix}. \tag{6a, 6b, 6c}$$

The sub-matrix $\mathbf{A_{TB}}$ is associated with aerodynamic generalized forces corresponding to Eq. (2); $\mathbf{A_{AE}}$ is related to both dynamic equilibrium equations and Eq. (5) but is independent of the turbulence. Dimensionless turbulence terms are simulated through a set of uncorrelated linear (autoregressive) filters (e.g., $d\hat{u} = -G_{1u}\hat{u}ds + G_{2u}dW(s)$). Matrix $\mathbf{C}$ is constant and linked to quantities $L_{\psi_j}\hat{u}(s), L_{\psi_j}\hat{w}(s)$; r is the maximum number of states and

$\mathbf{d} = \{\mathbf{0}_d, G_{2u}, G_{2w}, 0\}^{\mathrm{T}}$. The joint probability density function of the stationary process **Z**, $p(\mathbf{z})$, can be obtained from the solution of the following RFPE (e.g., [6]), as

$$-\sum_j \left[\sum_k A_{jk} z_k \frac{\partial p(\mathbf{z})}{\partial z_j} + A_{jj} p(\mathbf{z}) \right] + \pi \sum_{j,k} \left[\hat{C}_{j,k} p(\mathbf{z}) + q_{C,j} q_{C,k} \frac{\partial^2 p(\mathbf{z})}{\partial z_j \partial z_k} \right] = 0, \tag{7}$$

with $q_{C,j} = \left(\sum_k C_{jk} z_k + d_j \right)$, $\hat{C}_{j,k} = \left(2C_{jj} C_{kk} + 2C_{ij} C_{ji} + C_{ii}^2 + C_{jj}^2 \right)$.

4 DISCUSSION AND CONCLUDING REMARKS

This paper discusses the derivation of the steady-state solution to the dynamic buffeting response of a bridge in the presence of turbulence fields with uncertain span-wise correlation, restricted to a selected number of structural modes. The dynamic equilibrium equation was derived, which allows for the solution of the joint probability density function of an extended state vector. This vector includes not only the dynamic response but also the terms associated with the representation of the uncertain loss of span-wise correlation. The dependence of span-wise correlation L_{ψ_j} on the turbulence components (u, w) was represented through two compound random variables in $\mathbf{Z}_{TB}$. This assumption implies that, if the generalized input can be represented through a white noise, the process in Eq. (5) is inherently Gaussian and that the solution to the RFPE is unnecessary, since second-moment calculus is only necessarye. Nevertheless, this fact suggests that these "L_{ψ_j}" variables must also be normally distributed, which is perceived as possibly unrealistic.

The presentation of this work will also focus on some examples and a summary of the current studies, conducted to avoid this limitation by incorporating the variables as multiplicative terms in the state vector, i.e., a nonlinear system requiring truncation (Eqs. 5, 7).

ACKNOWLEDGEMENTS

This study has been supported by the US National Science Foundation (NSF), Grant CMMI 0600575 and, in part, by Northeastern University, Office of the Provost, Research and Scholarship Development Fund (2006-2007). Any findings and opinions are those of the author and do not necessarily reflect the views of the NSF.

REFERENCES

[1] N. P. Jones, R. H. Scanlan. Theory and full-bridge modeling of wind response of cable-supported bridges. *Journal of Bridge Engineering, ASCE*, **6**(6), 365-375, 2001.

[2] Y. L. Xu, L. D. Zhu. Buffeting response of long-span cable-supported bridges under skew winds. Part 2: case study. *Journal of Sound and Vibration*, **281**(3-5), 675-697, 2005.

[3] E. Ozkan, N. P. Jones. Evaluation of response prediction methodology for long-span bridges using full-scale measurements. Proceedings of the 11th Int. Conference on Wind Engineering, Lubbock, TX, USA, June 2-5, 2003, 1407-1414.

[4] L. Caracoglia. Influence of uncertainty in selected aerodynamic and structural parameters on the buffeting response of long-span bridges. *Journal of Wind Engineering and Industrial Aerodynamics*, **96**(3), 327-344, 2008.

[5] C. G. Bucher, Y. K. Lin. Stochastic stability of bridges considering coupled modes. *Journal of Engineering Mechanics, ASCE*, **114**(12), 2055-2071, 1988.

[6] M. Grigoriu. *Stochastic calculus. Applications in Science and Engineering*. Birkhäuser, Boston, MA, USA, 2002.