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SELF-EXCITED NONLINEAR RESPONSE OF A BRIDGE-TYPE CROSS SECTION IN POST-CRITICAL STATE

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Abstract: Long slender civil-engineering structures exposed to wind, especially decks of suspension bridges, their supporting elements, mast, towers and tall buildings are susceptible to vibrations under certain circumstances due to pure wind load action or due to special aeroelastic effects. Interaction between structural response and wind load, having the influence on the stability, is nowadays of basic importance in advanced technical design of such structures.

Finding the stationary points and determining stability characteristics of a system is an important undertaking in any attempt to understand or to modify its dynamical properties. We demonstrate the importance with an example describing the aeroelastic system with gyroscopic (non quadratic in the velocities) as well as with non-conservative terms. In the article we consider a non-linear system with two degrees of freedom representing an interaction of bending and torsion of a slender beam vibrating in a cross flow. The shape makes possible to separate principal effects and the coupling of aeroelastic modes is caused solely by the flow around the structure. The system is autoparametric and permits a semi-trivial solution of two types, one of which being unstable. Apart from that the response may be nontrivial in both phase coordinates.

Conditions of existence of stationary points and their types are investigated using primarily the Lyapunov function. The procedure presented is applicable for Hamiltonian, holonomic systems which are conservative, or nonconservative with certain limitations on the generalized forces. The singular points are be classified with respect to their asymptotically stable/unstable character together with adequate physical interpretation. Attention is paid to attractive and repulsive areas surrounding these points. Using this background several types of post-critical response in non-linear formulation will be presented, such as stable/unstable limit cycles with various ratio of amplitudes of both components, or quasi-periodic response processes having a form of symmetric or asymmetric beating effects with strong energy trans-flux between degrees of freedom.

1 INTRODUCTION

The most fundamental task of bridge design with regards to the aeroelasticity lies in the formulation of the self-excited forces and interaction with the response. From the theoretical point of view, this interaction leads to the origin of nonconservative forces contributing the to stiffness matrix and in the same time to the origin of forces influencing a damping matrix in linear and nonlinear way.

On the linear level, two parallel ways, each of them bringing some advantages, can be formulated. The duality of time and frequency domain formulation of the self-excited wind forces was and still is investigated [2], [1]. In the time domain formulation of self-excited forces on a bridge deck, indicial functions are adopted, see e.g. [3] among many. The analysis has revealed relatively considerable diversity of conclusions based on experimental studies as well as of the basic terminology. This is probably due to historical treatment of these problems in a number of branches as well as due to the existence of various types of instability domains and a number of bifurcation points of various properties. In the course of this period, however, the research has succeeded in understanding most principal characteristics of aeroelastic systems. In particular, repulsivity or attractivity of bifurcation has been studied and analysis of the behavior of the system in the post-critical state influenced by nonlinear terms has been accomplished, see [4] and other papers. They may lead to the stability loss, however, they can also act like stabilizing factors. They are able, at least for a limited period of time, to restitute one of lower types of stability after the structure has lost its exponential or asymptotic stability, e.g. [4].

Besides the pure theory, there has been collected a large quantity of experimental works related to this topic, see e.g. [5]. Any of these branches creates its own experimental conditions according to its needs with emphasis on its typical parameters resulting also in some theoretical incompatibility. It is uncertain which parameters are fundamental for the particular instability origin and which are only typical for the particular technical branch. A systematic research is needed to clarify the role of main individual parameters in the instability onset.

2 MODEL OF THE STRUCTURE AND BASIC PROPERTIES OF THE SYSTEM

We refer to the vibration of a slender prismatic bar in the combination of bending and torsion, circumvent by the flow of wind along its longer side. This shape makes possible to separate principal effects. It is symmetrical along two axes, its center of torsion is identical with its centroid and the effects of aerodynamic forces can be referred to the center of this section. The coupling of aeroelastic modes is caused solely by the flow around the structure.

The movement of the system can be generally described by a couple of differential equations:

$$\begin{aligned} \ddot{u} - 2\omega_{bu}\dot{u} + \omega_u^2 u &= K_m(1-\gamma_{uu}\dot{u}^2 - \gamma_{u\varphi}\dot{\varphi}^2)(b_{uu}\dot{u}^2 + b_{u\varphi}\dot{\varphi}^2) + \\ &+ K_m(1-\beta_{uu}\dot{u}^2 - \beta_{u\varphi}\varphi^2)(c_{uu}u + c_{u\varphi}\varphi) \\ \ddot{\varphi} - 2\omega_{b\varphi}\dot{\varphi} + \omega_\varphi^2\varphi &= K_J(1-\gamma_{\varphi u}\dot{u}^2 - \gamma_{\varphi\varphi}\dot{\varphi}^2)(-b_{\varphi u}\dot{u}^2 + b_{\varphi\varphi}\dot{\varphi}^2) + \\ &+ K_J(1-\beta_{uu}\dot{u}^2 - \beta_{u\varphi}\varphi^2)(-c_{\varphi u}u + c_{\varphi\varphi}\varphi) \end{aligned} \tag{1}$$

Basically only the third degree form is meaningful, as higher degrees are hardly to be identified experimentally and are problematic from physical point of view. The second and higher even degrees can be omitted with respect to the rectangular cross-section symmetry.

Coefficients K_J, K_m depend on the geometry and the wind speed. The parameters b_{ij}, c_{ij} are aeroelastic coefficients on the linear level, while b_u, b_φ are generalized damping coefficients. The coefficients β_{ij} and γ_{ij} represent scaling factors of the non-linear part and form a square diagonal or asymmetrical matrices respectively. The coefficient may be thought as general aeroelastic derivatives and should be identified experimentally.

3 STABILITY INSPECTION

The stability of the critical point of a system can usually be determined from a study of the corresponding linear system. However, for example, no conclusion can be drawn when the critical point is center of the corresponding linear system. Also, for an asymptotically stable critical point it may be important to investigate the region of asymptotic stability.

Conclusions about the stability of a critical point may be acquired by means of construction of a suitable auxiliary function, called Lyapunov function Φ. Its derivative Ψ, can be identified as the rate of change of Φ along the trajectory of the system that passes through the point of interest in the phase plane. There are no general methods of construction of Lyapunov function and often the judicious trial-and-error approach may be necessary. However, the use of some general properties of mechanical systems often gives good results. For instance, tools based on energy balance and the first integrals are usually very effective. If a system has several first integrals, the Lyapunov function can be written in a form of a linear combination of first integrals and possibly of their functions. The coefficients of these linear combinations, which can be considered as Lagrange multipliers, must be so determined as to get to the resulting function the properties of positive definiteness.

Let us assume, that the motion is allowed in rotation only. Though simplified, this assumption allows us to analyze some interesting properties and to understand the role of individual parameters in the system. We rewrite the system:

$$\dot{\varphi} = \psi\,;\, \dot{\psi} = -\left[2\omega_{b\varphi} - K_J b_{\varphi\varphi}\left(1 - \gamma_{\varphi\varphi}\psi^2\right)\right]\psi - \left[\omega_{\varphi}^2 - K_J c_{\varphi\varphi}\left(1 - \beta_{\varphi\varphi}\varphi^2\right)\right]\varphi \qquad (2)$$

There are three equilibrium points $\mathrm{P}_1 = (0,0)$, $\mathrm{P}_{1,2} = (\pm a, 0)$ where $a = ((c_{\varphi\varphi}K_J - \omega_{\varphi\varphi}^2)(c_{\varphi\varphi}K_J\beta_{\varphi\varphi}))^{1/2}$. The conclusion about the stability for critical points can be then drawn from the Lyapunov function and its derivative.

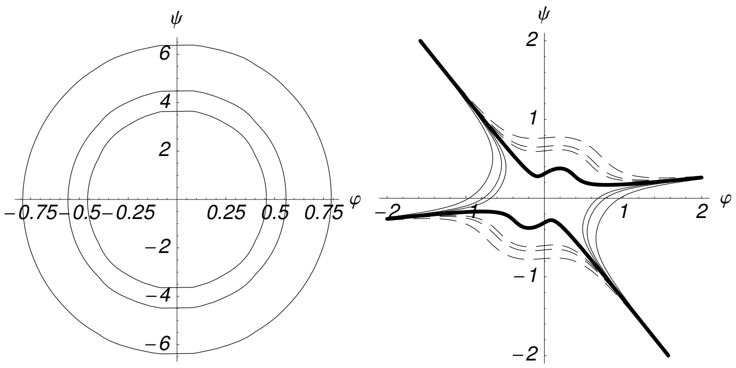

Figure 1. An example of Lyapunov function Φ (left) and its derivative Ψ (right) for the velocity $V = 10\,m/s$ in the neighborhood of critical point $(0,0)$. Dashed lines represent negative values.

The linear variational equation of the vector field in (2) lead to the characteristic equation and to the eigenvalues of the Jacobian matrix at critical points. Some analysis of the eigenvalues around specific critical points type agreed with the conclusions acquired by the Lyapunov direct method. More details are in the full version of the paper.

4 NUMERICAL AND EXPERIMENTAL SOLUTION

Several numerical solution and experiments has been done in order to investigate the system (1) behaviour changing the equation parameters and initial conditions. Example of experimental results is given on the Fig. 2. The time evolution of the solution is very sensitive to even slight change of them. Around critical points, the response amplitudes may rise dramatically, stabilize and rise again. Moreover, energy transmission between two modes is identified.

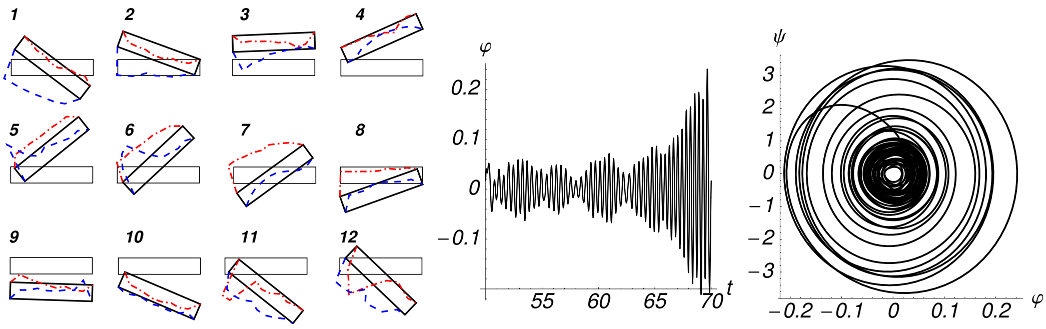

Figure 2. Left: Movement of the beam with large amplitudes during one period. The red (dash-dotted) lines are the pressures on the top surface of the beam. The blue (dashed) line represents the pressures on the bottom face of the body. Pressures pointing out of the body are positive. Right: Response and phase diagram from experiments with increasing velocity. Due to the influence of nonlinear terms, the oscillation has several bumps before the limit cycle oscillation starts.

5 CONCLUSIONS

The theoretical model of the bending-torsional flutter shows the considerable sensitivity of the system self-excited vibration. After the loss of stability of trivial solution the response in one degree-of freedom tends to stabilize itself in the form of approximate harmonic semi-trivial or nontrivial solution. The growth of this component after the loss of stability of trivial solution is prevented by a significant energy barrier. The system tends to escape to semi-trivial response into the other component. When this energy barrier has been overcome, however, the system loses stability and its response grows beyond all limits. This typical case has a number of intermediate steps and special states, the origin of which depends on the appropriate combination of parameters described in the text.

Lyapunov method is capable of providing an overview about behavior of the system under different conditions. The results obtained in the article depend however to a considerable extent on the parameters, which have to be identified from the measurements by complicated methods. These methods ought to be developed in near future.

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