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STRUCTURAL OPTIMIZATION OF THE FLUTTER PROBLEM IN SUSPENSION BRIDGES: AN ANALYTICAL FORMULATION

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1 INTRODUCTION

It is well known that long span suspension bridges are wind prone structures. They can be affected by a number of aerodynamic and aeroelastic phenomena such as vortex induced vibrations, galloping or flutter, amongst others. For decades, guaranteeing the safe performance of bridges against flutter has been one the main concerns of designers as that aeroelastic instability can be responsible for the complete destruction of the structure.

Therefore, a lot of work has been devoted to obtain bridge decks whose cross-section performs feasibly from the aerodynamic point of view. A clear example has been the wind tunnel testing campaign of the Great Belt Bridge where 1:80 scale section model tests were conducted in smooth and turbulent flow with the aim of evaluating the flutter performance of a series of candidate section designs [1].

However, it must be kept in mind that the behavior of a bridge under an oncoming wind flow is essentially a structural dynamics problem which can be written as:

$$\mathbf{M\ddot{u}} + \mathbf{C\dot{u}} + \mathbf{Ku} = \mathbf{f}_a = \mathbf{K}_a\mathbf{u} + \mathbf{C}_a\mathbf{\dot{u}}. \qquad (1)$$

Where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}_a$ is the vector containing the aeroelastic forces, $\mathbf{K}_a$ is the aeroelastic stiffness matrix and $\mathbf{C}_a$ is the aeroelastic damping matrix.

In the former expression, the vector of aeroelastic forces acting on the deck is obtained from:

$$\begin{pmatrix} D \\ L \\ M \end{pmatrix} = \frac{1}{2}\rho U^2 \begin{pmatrix} K^2P_4^* & -K^2P_6^* & -BK^2P_3^* \\ -K^2H_6^* & K^2H_4^* & BK^2H_3^* \\ -BK^2A_6^* & BK^2A_4^* & B^2K^2A_3^* \end{pmatrix}\begin{pmatrix} v \\ w \\ \varphi_x \end{pmatrix} + \\ +\frac{1}{2}\rho U^2 \begin{pmatrix} BKP_1^*/U & -BKP_5^*/U & -B^2KP_2^*/U \\ -BKH_5^*/U & BKH_1^*/U & B^2KH_2^*/U \\ -B^2KA_5^*/U & B^2KA_1^*/U & B^3KA_2^*/U \end{pmatrix}\begin{pmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{pmatrix}. \qquad (2)$$

It is clear that the vector of aeroelastic forces acting on the deck depends on the aerodynamic characteristics of the deck cross-section as the flutter derivatives appear in both $\mathbf{K}_a$ and $\mathbf{C}_a$. However, the left side of Eq. (1) depends on the structural characteristics of the structure.

Therefore, altogether with its aerodynamic characteristics it is possible to modify the structural properties of the bridge deck during the design phase in order to improve the bridge performance against flutter.
As a matter of fact, this work is based upon the assumption that a deck cross-section with a good aerodynamic behavior has been worked out and that, as a subsequent step, the structural characteristics of the bridge deck are going to be improved with the aim of obtaining a feasible performance against flutter. Thus the external shape of the deck cross-section is going to be kept constant along the design process so do the flutter derivatives.
Another key issue of this work is that a trial and error procedure changing the values of the structural properties of the deck to obtain improvements in the flutter response, that is, a study of parameters' variations is not applied. Instead of that, the rigorous mathematical formulation of the so-called optimum design or automated design is employed.

2 OPTIMUM DESIGN FORMULATION

Mathematically, the generic optimization problem can be written as

$$\min F(\mathbf{x}). \tag{3}$$

Which means that a certain objective function F, that depends upon a set of design variables $\mathbf{x}=(x_1,\ldots,x_n)$, being n the number of selected design variables, must be minimized, or maximized if that is the case.
Additionally, a set of design and behaviour constraints must be accomplished:

$$g_j(\mathbf{x}) \le 0 \quad j=1,\ldots,l \tag{4}$$

and

$$x_{i\min} \le x_i \le x_{i\max} \tag{5}$$

Where l is the total number of chosen constraints; and design variables x_i must be inside an interval of appropriated values.
In the present work the optimization problem to be solved is to obtain the minimum weight of a streamlined bridge deck compatible with a stipulated minimum flutter critical wind speed and a maximum allowed deck vertical displacement. Design variables may change continuously inside a range between a lower limit and an unbounded upper limit. Therefore the optimum design problem to be solved is:

$$\min F = A(e_1,\ldots,e_n) \tag{6}$$

Subject to:

$$e_{\min} \le e_i \le e_{\max} \quad i=1,\ldots,n \tag{7}$$

$$g_1:\ \frac{U_{f,cr}}{U_f} - 1 \le 0 \tag{8}$$

$$g_2:\ \frac{w_c}{w_{c,\max}} - 1 \le 0 \tag{9}$$

In the former expressions A is the area of the deck cross-section. The deck weight is directly related with the cross-section area; therefore, the minimum area deck corresponds with the minimum weight deck. When the suspension bridge deck is a box girder, then it is clear that the cross-section area is dependent upon the material thicknesses ($e_1,\ldots,e_n$). Material thicknesses must be included inside a certain range whose lower bound $e_{\min}$ is the minimum thickness due to manufacturing or constructive considerations. In practical applications the upper bound $e_{\max}$ can be neglected.

The behavior constraints to be satisfied are the following: The flutter speed U_f must be equal or higher than a critical value $U_{f,cr}$ established in the project requirements. Additionally, the vertical displacement of the span centre w_c must be equal or lower than a maximum bound $w_{c,\max}$ given by the project requirements or national codes for a certain set of actions. In a practical design case additional constraints should be included in the optimum design problem such as maximum allowable tresses or additional aerodynamic requirements like buffeting related ones.

The optimization method to be implemented for solving the design problem is the modified feasible directions method [2]. This is a gradient based method, therefore, objective function and constraints gradients must be supplied to the optimization algorithm. In spite of their popularity, finite differences are strongly discouraged from being used due to step size uncertainty, risk of mode order change and the enormous amount of the required computer time to evaluate, amongst others, the derivatives of the flutter speed with regard to the design variables [3]. The most efficient strategy developed to date for carrying out this task is the analytical evaluation of the derivatives. It must be borne in mind that the mentioned gradients must be evaluated repeatedly along the optimum design process which makes worthy the effort devoted to obtain the analytical formulation of the derivatives and their implementation in a computer code.

3 MAIN TASKS TO BE FORMULATED FOR THE AEROELASTIC OPTIMIZATION OF BRIDGES

In order to obtain the gradients of the two considered constraints, the derivatives of both the flutter wind speed and the deck displacements under dead and live loads with regards to the considered design variables must be obtained.

3.1 Analytical derivative of flutter speed

The analytical formulation for the evaluation of the derivative of the flutter wind speed with regards to a generic structural design variable x requires previously to carry out the analytical derivative of the natural frequencies and mode shapes [4] [5].

$$\frac{\partial\left(\omega_k^2\right)}{\partial x}=\frac{\boldsymbol{\phi}_k^T\cdot\left(\dfrac{\partial\mathbf{K}_{\text{nonlin}}}{\partial x}-\omega_k^2\cdot\dfrac{\partial\mathbf{M}}{\partial x}\right)\cdot\boldsymbol{\phi}_k}{\boldsymbol{\phi}_k^T\cdot\mathbf{M}\cdot\boldsymbol{\phi}_k} \tag{10}$$

$$\left(\mathbf{K}_{\text{nonlin}}-\omega_k^2\cdot\mathbf{M}\right)\cdot\frac{\partial\boldsymbol{\phi}_k}{\partial x}=\frac{\partial\left(\omega_k^2\right)}{\partial x}\cdot\mathbf{M}\cdot\boldsymbol{\phi}_k-\left(\frac{\partial\mathbf{K}_{\text{nonlin}}}{\partial x}-\omega_k^2\cdot\frac{\partial\mathbf{M}}{\partial x}\right)\cdot\boldsymbol{\phi}_k \tag{11}$$

Finally, deriving the state equation of the flutter problem the sensitivity of the flutter wind speed with regards to a generic design variable x is a real number which can be obtained from:

$$\frac{\partial U_f}{\partial x}=\frac{-\operatorname{Im}\left(\bar{g}_K h_{Ax}\right)}{\operatorname{Im}\left(\bar{g}_K g_U\right)} \tag{12}$$

3.2 Analytical derivative of the deck displacements

The analytical derivative of the deck displacements $\mathbf{u}$ with regards to a generic design variable x when dead or live loads are acting to the bridge can be obtained solving the following equation system:

$$\left[\mathbf{K}_{\text{lin}}+\mathbf{K}_{\text{G}}+\mathbf{IE}\left(\mathbf{K}_{\text{G}}^{\text{b}}\cdot\mathbf{u}\cdot\mathbf{w}^{\text{T}}\right)\right]\cdot\frac{\partial\mathbf{u}}{\partial x}=\frac{\partial\mathbf{p}}{\partial x}-\frac{\partial\mathbf{K}_{\text{lin}}}{\partial x}\cdot\mathbf{u}-\mathbf{IE}\left(\mathbf{K}_{\text{G}}^{\text{b}}\cdot\mathbf{u}^{\text{b}}\cdot\frac{\partial\mathbf{w}^{\text{T}}}{\partial x}\cdot\mathbf{u}^{\text{b}}\right) \tag{13}$$

3.3 Set of design variables

The assumed design variables in the optimum design problem formulation have been the thicknesses of a box cross-section (Eq. (6) to (9)). However, careful reflection is worthy to demonstrate that those design variables are the core ones governing the bridge deck aeroelastic behavior.

4 CONCLUSIONS

- Flutter response of bridges relies in both their aerodynamic and structural characteristics. Therefore, modifications of the structural parameters of bridge decks can improve their aeroelastic performance.
- The use of optimum design techniques avoids tedious and sometimes erratic trial and error design processes and guarantees that once a optimum design has reached it is the best of all possible designs and that its performance is better than previous "non-optimal" designs.
- The use of finite differences for evaluation of the derivatives of the flutter speed can produce inaccurate and misleading results.
- This design technique has been successfully applied to the Messina Strait Bridge [6].

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