1. Objectives

The flow under a train can be simplified in order to study the effect of the wall made up by sleepers and ballast. The easiest configurations to carry out this work are those corresponding to two-dimensional fully developed flow, such as Couette or Hagen-Poiseuille flows, in which periodic boundary conditions can be imposed at the entrance and exit. The Couette flow shown in figure 1. has been chosen, because it is the easiest one, and besides represents better the physics of the flow below the train. Three sleepers are included, and the ground in between has the roughness corresponding to the ballast. The flow field is calculated for the configuration shown in figure 1; specifically, the velocity profiles for several positions are obtained, see figure 2. A $k-\omega$ closure model to simulate turbulence was used, and calculations were carried out with Fluent. The moving upper wall is assumed to be smooth in most simulations, although other values of the roughness have also been considered for comparison. In the lower wall there are alternating regions of sleepers and rough walls simulating the ballast. The sleepers are supposed to be smooth. For the roughness of the ballast region two cases have been considered in this preliminary analysis: an equivalent roughness of $k=0.04$ m and smooth surface.

The average velocity profile is estimated and this is fitted to a logarithmic profile, from which the average roughness is obtained. The calculations are repeated for several configurations of the sleepers, and the obtained roughness is compared with the values found in the literature, given in [Jiménez, 2004].

In order to estimate the validity of the whole profile across the gap, an analytic solution of the turbulent Couette flow (using the equivalent roughness for the lower wall) has been calculated. This analytic solution is obtained using either the $k-\varepsilon$ or $k-\omega$ closure model; and it turns out to be the same, independently of which of the two models is used. This solution is the same one obtained by von Karman (1937), by postulating the existence of homologous turbulence. Its validity is discussed by Bech and Andersson (1996), which carried out comparisons with databases originating from a direct numerical simulation. The comparison between the analytic solution and the average velocity profiles is good.
Data have been provided by DB and SNCF, according to table 1.

**Table 1** Data used for the configuration of figure 1.

<table>
<thead>
<tr>
<th>Set of parameters</th>
<th>h (m)</th>
<th>L (m)</th>
<th>Ls (m)</th>
<th>K (m)</th>
<th>V (km/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deutsche Bahn</td>
<td>0.40</td>
<td>0.38</td>
<td>0.62</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.127</td>
<td></td>
<td></td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>SNCF</td>
<td>0.40</td>
<td>0.38</td>
<td>0.6</td>
<td>0.04</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.29</td>
<td></td>
<td></td>
<td>0.02</td>
<td></td>
</tr>
</tbody>
</table>

2. Results

A typical solution of the previous problem is given in figure 2, where the velocity profiles, corresponding to equidistant points along the channel, are shown. Details of the recirculation region after the step are also shown.

The law of the wall for a fully rough wall can be expressed, in terms of wall units, as:

\[ u^+ = 2.5 \ln \varepsilon^+ + B \]  

(1)
where \( u^* = u / u^* \), \( z^* = z u^* / \nu \), \( u^* \) is the friction velocity, \( z \) is the distance to the wall, and \( k_s \) is the equivalent roughness where \( \nu = 0.14 \times 10^{-4} \text{ m}^2/\text{s} \) is the kinematic viscosity of air, and the constant is given by:

\[
B = 8.5 - 2.5 \ln \left( \frac{k_s u^*}{\nu} \right)
\]

(2)

where the friction velocity \( u^* \) is obtained numerically. The value of \( k_s \) is then obtained from equation (2), by fitting to the average profile, and is given in table 2. In that table is also given the value of the roughness as used in geophysical applications, that is: \( z_0 = k_s / 30 \).

Besides the logarithmic law, that is valid only near the lower wall, a 2D analytical solution valid for the full region, will be useful to estimate the flow profile and for comparison with numerical results. This analytical solution can also be used to make a first estimation of the value of the shear stress which is a parameter that is most probably related to the raise of the ballast. Each surface, both upper and lower, can be either rough or smooth. This solution will be useful for comparison with previous results, by substituting the sleepers and ballast by the surface with the equivalent roughness. It can be also of interest to determine the origin of \( z \). Its deduction has been made using the \( k-\varepsilon \) model, but an identical result can be obtained with the \( k-\omega \) model. According to this model the shear stress \( \rho u^* \) will be uniform across the gap; and the velocity will be given by:

\[
\begin{align*}
\frac{u^*}{u^*} &= 2.5 u^* \\
&\left\{ \ln \left( \frac{\sin \frac{\pi z}{2h}}{\frac{\pi z_{01}}{2h}} \right) - \ln \left( \cos \frac{\pi z}{2h} \right) \right\} \\
&= V \\
&2.5 \left( \ln \frac{2h}{\pi z_{01}} + \ln \frac{2h}{\pi z_{02}} \right)
\end{align*}
\]

(3)

where the value of the friction velocity \( u^* \) is given by

\[
\frac{u^*}{u^*} = \frac{V}{2.5 \left( \ln \frac{2h}{\pi z_{01}} + \ln \frac{2h}{\pi z_{02}} \right)}
\]

(4)

If a wall is smooth the value of \( z_0 \) will be given by:

\[
z_0 = 0.113 \frac{V}{u^*}
\]

(5)

<table>
<thead>
<tr>
<th>Table 2: Equivalent surface roughness.</th>
</tr>
</thead>
<tbody>
<tr>
<td>K (m)</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>DB track</td>
</tr>
<tr>
<td>0.02</td>
</tr>
<tr>
<td>SNCF track</td>
</tr>
<tr>
<td>0.02</td>
</tr>
</tbody>
</table>

Comparisons between the analytical results for smooth walls with experiments and classical models for turbulent Couette flows by Lund and Bush, (1980) show good agreement; although these comparisons are not directly related to the application and objectives of this project, they can be useful to validate the analytical model proposed for a turbulent Couette flow. Velocity
measurements between the gap and ground have been carried out by other partners within the DeuFrako-AOA project, however these results have not been published yet, and comparisons will be made in future works.

3. Conclusions
A procedure has been implemented to estimate the equivalent roughness of the ground below the train, which consists of both ballast and sleepers. The calculated values of the equivalent roughness are similar and have similar tendencies to others found in the literature: Leonardi et al, (2004) and Jiménez (2004).

The flow fields obtained with the equivalent roughness and those obtained with the real ground (ballast and sleepers) show similar behaviours at a certain distance above the ground. The obtained average profile is compared with the law of the wall and the analytic solution for the equivalent roughness. The origin of the vertical coordinate is chosen so that a best fit is obtained between the law of the wall and the average profile near the ground. This origin of the vertical coordinate turns out to be at the surface of the ballast.

An analytical solution of the turbulent Couette flow (using the equivalent roughness for the lower wall) has been calculated; it reproduces exactly the numerical solutions, except near the wall, and at distances of the order of or smaller than the equivalent roughness, where the numerical solution fails. This analytical solution can also be used to make a first estimation of the value of the shear stress, which is a parameter that is most probably related to the raise of the ballast. This analytical solution shows a reasonable agreement with experimental data of velocity obtained in the gap between the train and the ground. The comparison between the analytic solution and the average velocity profiles is good.

The influence of the configuration on the obtained values of the equivalent surface roughness is analyzed. The following parameters have been changed: height of the gap, Reynolds number (by changing the velocity of the upper wall), roughness of the upper wall (in the initial configuration this wall is supposed to be smooth). The equivalent roughness seemed to be insensitive, to variations of these parameters.

The influence of the turbulence closure procedure on the results has been examined and different equivalent roughness are obtained, depending on whether the k-ε or k-ω closure procedure is used. Presumably, this is due to the way in which the two methods calculate the recirculation behind the sleepers. If the calculations are performed using an equivalent roughness the solutions obtained with the two closure methods are the same, and equal to the exact solution.

Some of the equivalent roughness obtained are quite large, only somewhat smaller than the gap, and this may create a conflict in some commercial codes The calculations, carried out with Fluent and the equivalent roughness really confirm that for mesh sizes less than half the size of the equivalent roughness the solution obtained differs considerably from the exact solution.
References:


