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A NEW GENERAL 3DOF QUASI-STEADY AERODYNAMIC INSTABILITY MODEL.

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Abstract

This paper proposes a three degrees of freedom (3DOF) quasi-steady aerodynamic model and instability criterion for a bluff body which is uniform along the length axis. The model and criterion has been developed in the frame of investigating aerodynamic instability of cables due to ice accretions but can generally be applied for aerodynamic instability prediction for prismatic bluff bodies. The 3DOF, which make up the movement of the model, are the displacements in the XY-plane and the rotation around the bluff body's rotational axis. The proposed model incorporates inertia coupling between the three degrees of freedom and is capable of estimating the onset of aerodynamic instability for changes in drag, lift and moment, which is a function of wind angle of attack (α) in relation to the x-axis of the bluff body, Reynolds number and wind angle (ϕ) in relation to the length axis of the bluff body. Further more the model is capable of predicting an estimate for the structural damping needed for avoiding instability of the bluff body.

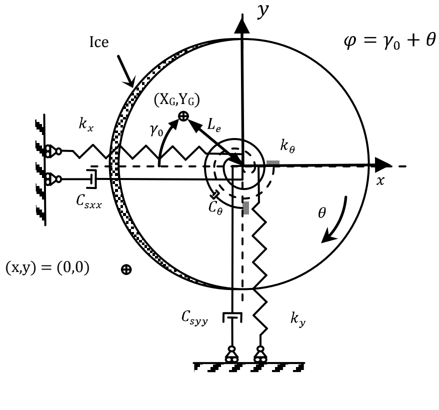


Figure 1. Definition of system.

x = displacement direction.
y = displacement direction.
θ = rotation direction.
γ_0 = angle offset for mass centre.
L_e = mass centre offset.
k_x = structural stiffness in x direction.
k_y = structural stiffness in y direction.
k_θ = structural damping in θ direction.
C_{sxx} = structural damping in x direction.
C_{syy} = structural damping in y direction.
C_θ = structural stiffness in θ direction.
(X_G,Y_G) = mass centre.

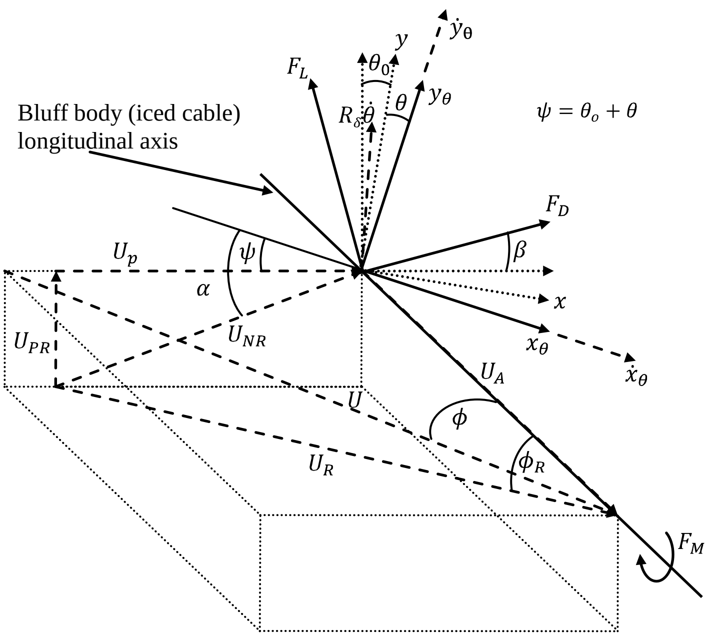
1 INTRODUCTION

The main purpose for models which investigates aerodynamic stability is to predict when instability occurs. During the last 80 years a number of models have been proposed and over the last couple of years aerodynamic damping, as a driving force for vibration, has received some renewed attention. Den Hartog proposed his stability criterion in 1932 Ref. [1] for a 1DOF system, which was made for a bluff body with an aerodynamic lift coefficient formulated as a function of wind angle of attack in relation to the surface of the bluff body. Later, 1962, Davenport proposed an expression for the aerodynamic damping in the along wind and the transverse wind direction of a cylinder Ref. [2]. In 1981 Martin et al. proposed the instability criteria which now is known under the name of "Drag instability" Ref. [3]. Up to that point all expressions for the aerodynamic damping were special cases, which should be applied individually. In 2006 a unified approach to damping and drag/lift instabilities was proposes by Macdonald and Larose Ref. [4] for a 1DOF system which was later extended to a 2DOF system. This general quasi-steady 2DOF instability model is able to estimate the structural damping needed for avoiding instability for a bluff body moving in the XY-plane. The 2DOF model is capable of predicting the special cases which was proposes earlier, but lack the possibility of predicting inertial coupling for a bluff body with a mass centre offset from its rotational axis.

The new 3DOF quasi-steady model proposed in this paper is capable of predicting the same levels of damping as the 2DOF model proposes by Macdonald et al, furthermore it is capable of predicting torsional instability in conjunction with the movement in the XY-plane.

2 THE MODEL

The bluff body model is based on a section model approach and developed for a cable with a thin ice accretion. The application of this model can be expanded to any geometry of a bluff body subjected to aerodynamic instability. The position of the ice accretion is described in the x-y coordinate system of the cross-sectional plane of the still body. If rotation is induced the angle θ describes the magnitude of rotation around the length axis of the bluff body.



$x_\theta, \dot{x}_\theta$ = displacement, velocity.
$y_\theta, \dot{y}_\theta$ = displacement, velocity.
θ_0 = steady wind angle of attack
$\theta, \dot{\theta}$ = structural rotation, angular velocity.
α = wind angle of attack, cable surface.
β = angle of rotation, relative wind.
ϕ = wind angle of attack, cable length axis.
F_L = lift.
F_D = drag.
F_M = moment.
R_δ = leading edge length.
U = mean wind velocity.
U_R = relative wind velocity.
U_{PR} = projected relative vertical wind velocity.
U_p = projected relative horizontal wind velocity.
U_{NR} = normal projected relative wind velocity.
U_A = along axis wind velocity.

Figure 2. Schematic model of cable section with ice accretion.

Figure 1 and Figure 2 show the bluff body system which is illustrated as a cable with a thin ice accretion.

3 FORMULATION OF EQUATIONS

3.1 Equations of motion

The equation of motions (EOMs) is given in Eq. (1)-(3).

$$m\ddot{x} + C_{sxx}\dot{x} + k_x x + mL_e\left(cos(\varphi)\,\dot{\theta}^2 + sin(\varphi)\,\ddot{\theta}\right) = F_x \tag{1}$$

$$m\ddot{y} + C_{syy}\dot{y} + k_y y + mL_e\left(-sin(\varphi)\,\dot{\theta}^2 + cos(\varphi)\,\ddot{\theta}\right) = F_y \tag{2}$$

$$J\ddot{\theta} + C_{s\theta\theta}\dot{\theta} + k_\theta\theta + mL_e\left(sin(\varphi)\,\ddot{x} + cos(\varphi)\,\ddot{y} + L_e\ddot{\theta}\right) = F_\theta \tag{3}$$

F_x, F_y and F_θ are the external aerodynamic forces which are presented in Eq. (4) to Eq. (6) for small initial displacements, where $\alpha = -(\psi + \beta)$ and φ are given in Figure 1.

3.2 Aerodynamic forces

$$F_\theta = \frac{1}{2}\rho U_r D C_M(\alpha, Re, \phi) \tag{4}$$

$$F_x = \frac{1}{2}\rho U_r D(C_D(\alpha, Re, \phi)\,cos(\alpha) + C_L(\alpha, Re, \phi)\,sin(\alpha)) \tag{5}$$

$$F_y = \frac{1}{2}\rho U_r D(C_L(\alpha, Re, \phi)\,cos(\alpha) - C_D(\alpha, Re, \phi)\,sin(\alpha)) \tag{6}$$

3.3 Aerodynamic damping

Eq. (7) gives the aerodynamic damping matrix calculated for small initial displacements ($\cos(\theta) = 1,\ \sin(\theta) = 0$). The values of the aerodynamic damping matrix C_a are found by applying an approach presented and discussed in Ref. [4]. Eq. (8) gives the total damping matrix containing both structural and aerodynamic damping.

$$C_a = -\begin{bmatrix} \frac{\partial F_x}{\partial \dot{x}} & \frac{\partial F_x}{\partial \dot{y}} & \frac{\partial F_x}{\partial \dot{\theta}} \\ \frac{\partial F_y}{\partial \dot{x}} & \frac{\partial F_y}{\partial \dot{y}} & \frac{\partial F_y}{\partial \dot{\theta}} \\ \frac{\partial F_\theta}{\partial \dot{x}} & \frac{\partial F_\theta}{\partial \dot{y}} & \frac{\partial F_\theta}{\partial \dot{\theta}} \end{bmatrix}_{\dot{x}=\dot{y}=\dot{\theta}=0} \tag{7}$$

The stability criteria for the 3DOF system are based on a Taylor expansion of the aerodynamic forces about $\dot{x} = \dot{y} = \dot{\theta} = 0$ which results in a static and a dynamic wind force. Furthermore it is assumed that all higher order terms in the EOMs are negligible. These assumptions correspond to the instant where a vibration event is initiated.

$$C_D = C_s + C_a = \begin{bmatrix} C_{sxx} & 0 & 0 \\ 0 & C_{syy} & 0 \\ 0 & 0 & C_{s\theta\theta} \end{bmatrix} + \begin{bmatrix} C_{axx} & C_{axy} & C_{ax\theta} \\ C_{ayx} & C_{ayy} & C_{ay\theta} \\ C_{a\theta x} & C_{a\theta y} & C_{a\theta\theta} \end{bmatrix} = \begin{bmatrix} C_{xx} & C_{xy} & C_{x\theta} \\ C_{yx} & C_{yy} & C_{y\theta} \\ C_{\theta x} & C_{\theta y} & C_{\theta\theta} \end{bmatrix} \quad (8)$$

With these assumptions it is possible estimate the stability of the 3DOF system by rewriting the equations of motions (Eq. (1)-(3)) into state space and solving the eigenvalue problem which can be obtained here from.

3.4 Prediction of instability

Using aerodynamic data obtain from previously made wind tunnel test Ref. [5] shows that there is good agreement between the experimental observed instability and the range predicted from the suggested model.

Figure 3 gives the aerodynamic input data, used by the here presented new model, to calculate the instability ranges shown in Figure 4. Values less than zero indicate the ranges of instability predicted by the model. The stable ranges are 0° - ~25°, ~45° - ~70° and ~135° - ~170°. In Figure 4 it is seen that the new model is capable of predicting instability over a wide range of wind angle of attack which is a combined effect of drag lift and moment which previous model cannot.

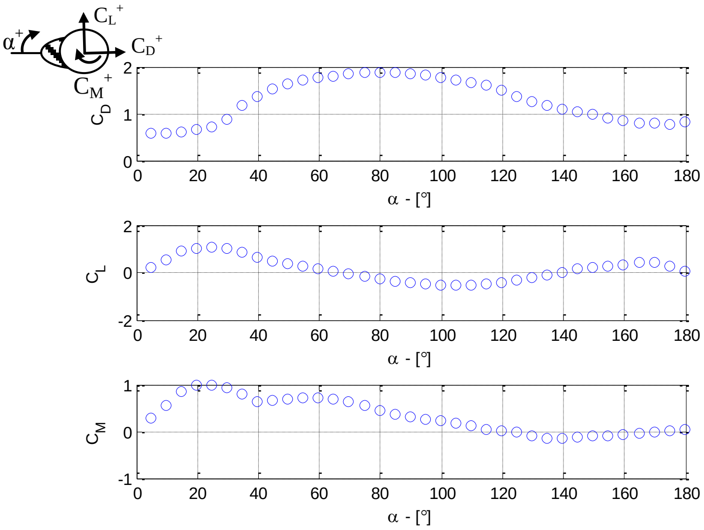


Figure 3. Aerodynamic coefficients presented by Ref. [5] and transformed into the models coordinate system.

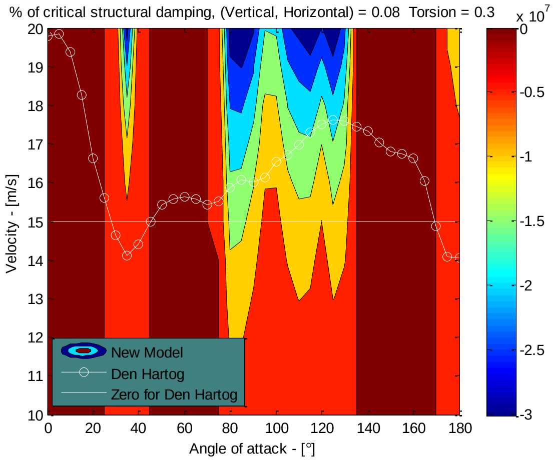


Figure 4. Range of predicted instability for the combined Drag, Lift and moment and $\gamma_0 = 0°$.

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