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ANAYTICAL MODELS FOR THE LIFT AND DRAG FORCES ON ROTATIONALLY OSCILLATING CYLINDERS

Isam Janajreh* **and Muhammad Hajj†**

*MIT Visiting Professor
Massachusetts Institute of Technology, Cambridge, MA, USA
e-mail: janajreh@mit.edu ,

† Professor
Department of Engineering Science and Mechanics, Virginia Tech, Blacksburg, VA
e-mail: mhajj@vt.edu

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1 INTRODUCTION

Oscillating drag and lift forces on circular cylinders are directly related to the vortex shedding pattern in their wakes. In different applications, the interest would be in reducing these forces, reducing induced-vibrations, or augmenting the lift component. One or more of these objectives can be achieved through forced rotational oscillations of the cylinder. In this work, analytical models are developed for the lift and drag forces on a rotationally oscillating cylinder. Three cases at Re =100, including stationary, lock-on and non lock-on cases are considered. Numerical simulations are performed to generate a database from which parameters for the developed models are determined. Amplitude and phase measurements from higher-order spectral parameters are matched with approximate solutions of the models to characterize the nonlinearities in the model and determine the corresponding parameters.

2 NUMERICAL SIMULATION

Direct Numerical Simulations of the unsteady incompressible Navier-Stokes equations for different cases of the flow over a rotationally oscillating circular cylinder were performed. All simulations were performed at $\mathrm{Re} = \rho U_\infty D / \mu =$ 100. The computational domain extended 5 cylinder diameters upstream, 10 diameters cross-stream on each side and 20 diameters downstream. The domain was staggered by multiple blocks with quadratic cell type mesh, in order to provide more faces and to enhance the cell communication and computational accuracy. The cylinder wall was padded with a boundary layer mesh to accurately capture the viscous layer. The first cell thickness is 0.0002D and with a linear growth rate of 1.05.

Imposed cylinder rotations were determined by two parameters, namely, the nondimensional amplitude, $\Omega = \dot{\theta}_{\max} D/2U_\infty$ where $\dot{\theta}_{\max}$ is the maximum angular velocity, and frequency $St_f = f_f d/U_\infty$, where f_f is the dimensional forcing frequency.

3 RESULTS

The flow pattern in the wake of the cylinder and the time series and spectra of the lift and drag coefficients for the three considered cases are presented in figures 1, 2, and 3. The wake in the stationary case shows the typical pattern of a von-Karman vortex street. The time series of the drag and lift are mostly sinusoidal with a major frequency in the lift corresponding to the vortex shedding frequency and a major one in the drag corresponding to twice of the vortex shedding frequency. These characteristics can also be realized from the spectra presented in figure 1-c. The additional third harmonic in the lift spectrum points to the capability of its modeling with a van der Pol oscillator and the clear phase relation between the lift and drag points to the possibility of relating them.

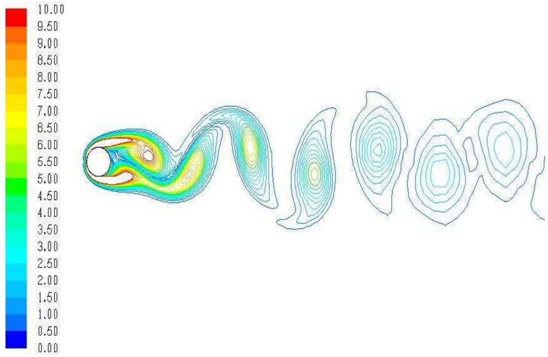

Figure 1-a: Vortex shedding pattern in the wake of a stationary cylinder, Re = 100

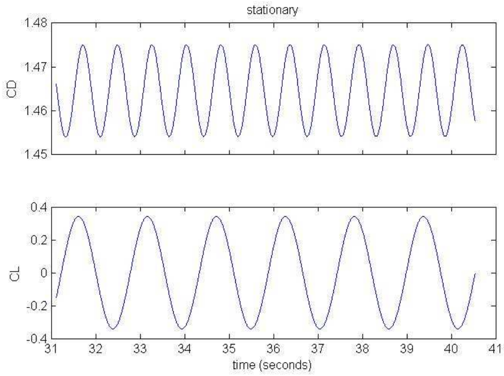

Figure 1-b: Time Series of the drag and lift on a stationary circular cylinder, Re = 100

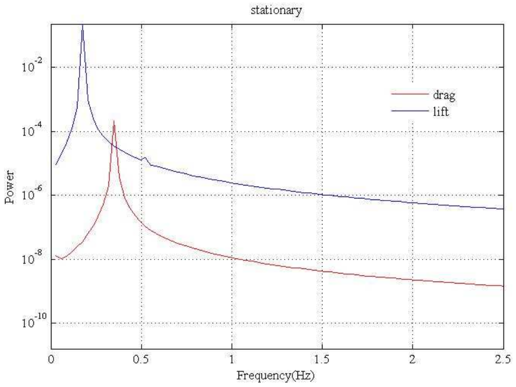

Figure 1-c: Spectra of the lift and drag on a stationary circular cylinder, Re =100

The flow pattern in the wake of the rotationally oscillating cylinder in the lock-on case and the corresponding time series and spectra of the lift and drag coefficients are presented in figure 2.

The results clearly show a similar vortex shedding pattern to the case of stationary cylinder with a significant increase in values of both drag and lift coefficients. The spectral distributions shows the vortex shedding frequency and its harmonic which points to modeling the lift in this case by a forced van der Pol equation under a resonance condition.

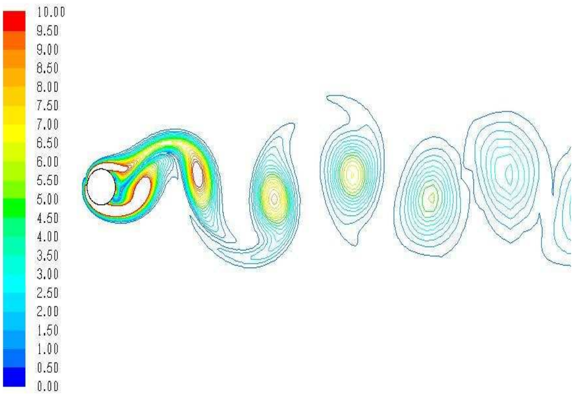

Figure 2-a: Vortex shedding pattern in the wake of a rotationally oscillating cylinder, lock-on case, Re = 100

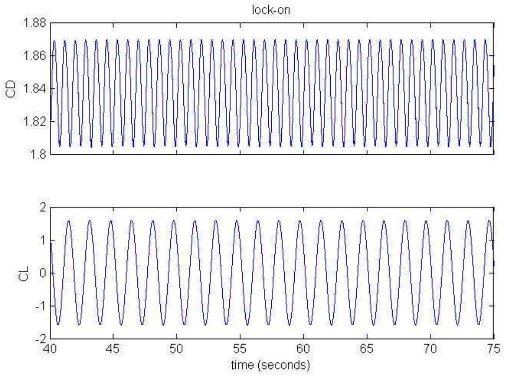

Figure 2-b: Time Series of the drag and lift on a rotationally oscillating cylinder, lock-on case, Re = 100

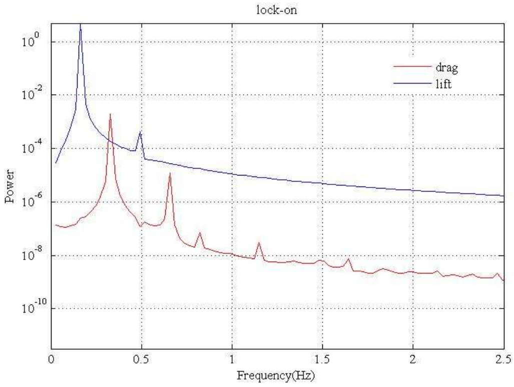

Figure 2-c: Spectra of the drag and lift on a rotationally oscillating cylinder, lock-on case, Re = 100

The flow pattern in the wake of the rotationally oscillating cylinder in the non lock-on case and the corresponding time series and spectra of the lift and drag coefficients are presented in figure 3. The results clearly show a different vortex shedding pattern than the case of stationary cylinder. The time series and spectra point to low-frequency variations in both the lift and drag coefficients. The lift in this case is also modeled with a forced van der Pol equation under non resonance conditions and the drag is related to the lift through a quadratic relation.

4 CONCLUSIONS

In this work, analytical models for the lift and drag forces on a rotationally oscillating cylinder are developed. Three flows at Re =100, including stationary, lock-on and non lock-on cases have been considered. Numerical simulations of the flow fields have been performed to generate the databases required to develop the models. The lift on the stationary cylinder, whose oscillations contain a major component at the vortex shedding frequency and a smaller one at its third harmonic, is modeled by the van der Pol equation. The corresponding drag coefficient contains the second harmonic of the vortex shedding frequency component and is, thus, modeled as a quadratic function of the lift. In the full paper, it will be shown how the form of this function can be determined from the phase of the cross-bispectrum between the drag and lift time series. The lift on the rotating cylinder is modeled by a forced van der Pol equation under different resonance conditions to represent the lock-on and non lock-on regimes. The drag is again modeled as a quadratic function of the lift.

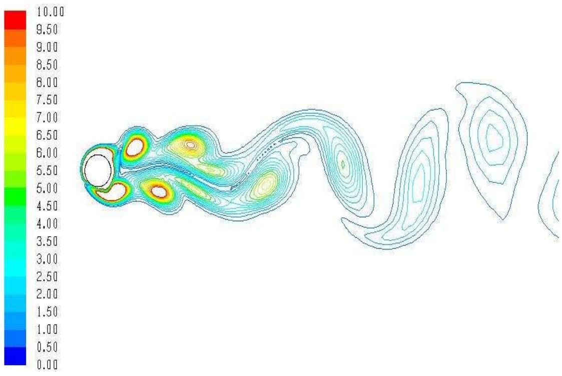

Figure 3-a: Vortex shedding pattern in the wake of a rotationally oscillating cylinder, non lock-on case, Re = 100

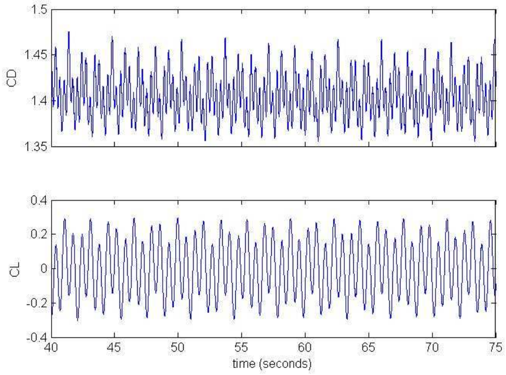

Figure 3-b: Time Series of the drag and lift on a rotationally oscillating cylinder, non lock-on case, Re = 100

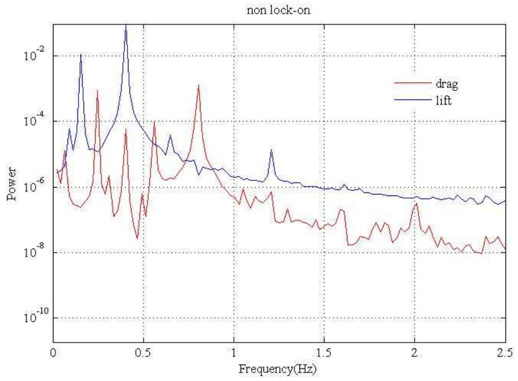

Figure 3-c: Spectra of the drag and lift on a rotationally oscillating cylinder, non lock-on case, Re = 100

REFERENCES

This work is based on previous work and extensive references that will be given in the full paper.