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STATISTICAL SYMMETRIES OF FLOW-INDUCED FORCES ON CYLINDERS

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1 INTRODUCTION

A very popular problem in experimental fluid-dynamics is constituted by the measurements of the fluid-induced forces acting on cylinders subjected to a cross flow produced in a wind tunnel. The global forces acting on the cylinder are originated by the unbalance of the pressure field acting on its surface, which, in turns, is related to the motion field of the fluid around it. The pressure field is randomly variable in time and space due to instabilities in the boundary layer and in the wake (if the Reynolds number in large enough) and due to the possible presence of turbulence in the incoming flow.

Typical experimental setups involve cylinders oriented in such a way to have a symmetry plane coincident to the symmetry plane of the test facility (e.g. vertical symmetric cylinders in a boundary-layer wind tunnel oriented at zero angle of attack). In such a condition, the pressure field is expected to fulfil some statistical symmetry condition, while, the statistics of the global forces should comply with some consequent restriction. It is known, for example, that, in this condition, the cross-wind force has zero mean value at any spanwise location along the cylinder and is uncorrelated with the along-wind force. In practise, these results are not rigorously achieved due to experimental imperfections or insufficient sampling, leading to a small lack of symmetry that is usually ignored (e.g. the cross-wind force is depurated from the small mean value that was actually measured).

Unfortunately, as it is shows in a case study presented herein, some statistical quantities often used in fluid-dynamics may be very sensitive to an even small lack of statistical symmetry of the pressure field, leading to the misevaluation of the experimental results.

With the objective of clarifying the mentioned problem, the present paper introduces the concept of statistically-symmetric random process presenting a definition in which the symmetry constraint is imposed on the characteristic functional of the process. Starting from such a definition, the symmetry constraint on the statistical moments of the pressure field and of the forces-per-unit-length (FPUL) are derived. It is demonstrated that the covariance eigenfunctions of the pressure field are necessarily symmetric or antisymmetric and that the statistical moments of the FPUL must fulfil the relationship $E[f_x^m f_y^n m_z^p]=0$ for $n+p$ odd, f_x, f_y, m_z being the along-wind force, the cross-wind force and the torsional moment. Also, it is demonstrated that the covariance eigenfunctions of the FPUL $\mathbf{f}(z)=[f_x(z)\ f_y(z)]^T$, z being the span-wise abscissa, are oriented along the direction of the flow or orthogonal to it.

Finally, a numerical procedure aimed at correcting measured data to impose a rigorous second-order statistical symmetry is presented and applied on pressure measurements carried out during a wind-tunnel test.

2 FLOW-INDUCED FORCES ON A PRISMATIC BODY: CASE STUDY

The present paragraph has mainly a motivational purpose and is intended as a summary of some notorious symmetry-related issues relevant in wind-tunnel tests. Some statistical properties of the flow-induced forces measured on a prismatic model tested in a boundary-layer wind tunnel are evaluated and discussed form a qualitative point of view, observing some well-known and intuitive behaviours, as well as some unusual an counterintuitive ones, which though they were reported in the technical literature, did not received any theoretical explanation. The model (Fig. 1) representing a square-base high-rise building, is instrumented by 500 pressure taps uniformly distributed on its lateral surface and simultaneously acquired in such a way to estimate the instantaneous pressure field acting on the model. The values measured by all the pressure channels are referred to a common reference pressure corresponding to the wind-tunnel static pressure at the test section. The referred test was carried out at the Shimizu Corporation Laboratories and further details can be found in [1].

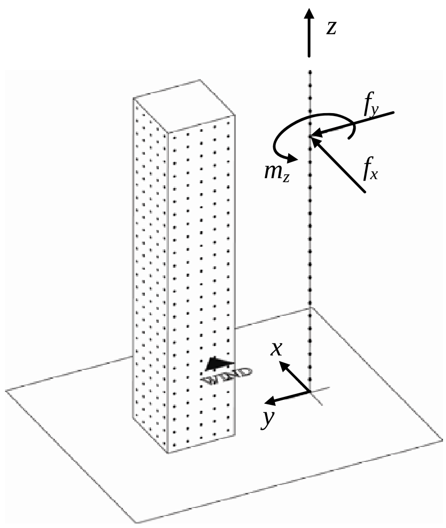

Figure 1. Case study: experimental setup.

Let f_x and f_y be, respectively, the along-wind and cross-wind FPUL acting on the model, evaluated integrating the pressure field at each instrumented level. The statistics of the FPUL are estimated from the data, assuming that the pressure field is a stationary and ergodic random process, by averaging the instantaneous values measured throughout the whole test.

Figure 2a shows the mean value of the along-wind FPUL, $\mu_{f_x}(z_j)$ (nondimensionalised by $0.5\rho bU^2$, ρ being the air density, b the side of the cylinder and U the incoming mean wind velocity), evaluated at the 25 instrumented levels z_j (j=1,…,25) of the model averaging a number of samples ranging form 1 to 3×10^4; as the number of samples increases, the plots quickly converge to stable values, which are greater for higher levels due to the mean-velocity profile in the boundary layer. Exception to this behaviour are levels j=1 and j=25, respectively, at the base and at the top of the cylinder. Figure 2b shows the mean value of the cross-wind FPUL, $\mu_{f_y}(z_j)$, nondimensionalised by the along-wind FPUL at the corresponding level; as the number of samples increases, μ_{f_y} rapidly converge to levels constituting a few percent of the along-wind forces. Such values are interpreted as experimental errors due to imperfections in the instrument calibration or in the wind-tunnel flow characteristics and are often corrected by fine-tuning the experimental setup. Figures 2c,d,e show the root mean square (rms) of along-wind (σ_{f_x}, Fig. 2c) and cross-wind (σ_{f_y}, Fig. 2d) FPUL, as well as their correlation coefficient ($\rho_{f_x f_y}$, Fig. 2e), versus the number of samples included in the average. The rms of both the FPUL are nondimensionalised by the mean value of the along-wind FPUL at the corresponding level. All the three mentioned quantities have a fast convergence to statistically stable values; the correlation coefficients converge to small values that, again, are interpreted as consequences of experimental imperfections.

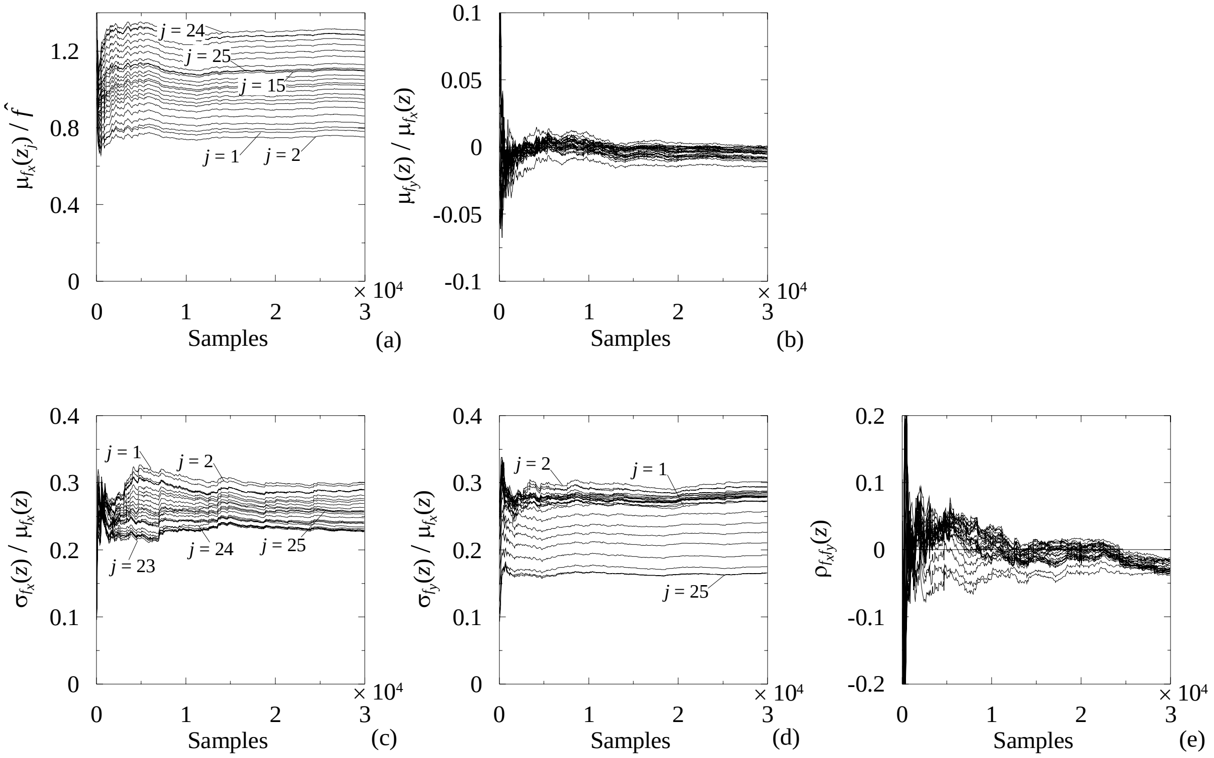

Figure 2. Single-point statistics of FPUL vs number of samples. Mean along-wind (a) and cross-wind (b) FPUL; Rms along-wind (c) and cross-wind (d) FPUL; correlation coefficient along-wind and cross-wind FPUL (e).

The quantities described above are very basic tools for the analysis of flow-induced forces, their derivation from the data is straightforward, and it is not surprising to see some evident consequence of symmetry (μ_{f_x}≃0, $\rho_{f_x f_y}$≃0). However, for other statistical quantities widely used in practice, the situation is rather more complicate. As an example, let us consider the vector of the FPUL $\mathbf{f}(z) = [f_x(z)\ f_y(z)]^{\mathrm{T}}$ and its covariance function $\mathbf{C_{ff}}(z_1,z_2)=\mathrm{E}[\mathbf{f}(z_1)\mathbf{f}(z_2)^{\mathrm{T}}]$, E[•] being the statistic average operator. Let $\boldsymbol{\phi}_k(z)$ be an eigenfunction of $\mathbf{C_{ff}}$, i.e. a solution of the integral equation:

$$\int_0^h \mathbf{C_{ff}}\left(z_1,z_2\right)\boldsymbol{\phi}_k\left(z_2\right)\mathrm{d}z_2 = \lambda_k \boldsymbol{\phi}_k\left(z_1\right) \qquad z_1 \in \left[0,h\right], \quad k = 1,2,... \tag{1}$$

and λ_k the corresponding eigenvalue. The eigenfunctions $\boldsymbol{\phi}_k(z)$ are functions in $\mathbb{R}^2$ and can be used as a base to represent $\mathbf{f}(z)$ as:

$$\mathbf{f}(z) = \boldsymbol{\mu}_{\mathbf{f}}(z) + \sum_{k=1}^{\infty} \boldsymbol{\phi}_k(z)\, x_k \tag{2}$$

where $\boldsymbol{\mu}_{\mathbf{f}}(z)$=E[$\mathbf{f}(z)$] is the mean value of $\mathbf{f}(z)$ and x_k are uncorrelated random variables whose variance is provided by the eigenvalues λ_k. The spectral representation expressed by Eq. (2) is referred to as Proper Orthogonal Decomposition (POD) and is widely employed in many scientific fields including fluid-dynamics and bluff-body aerodynamics [1,2]. The eigenfunctions $\boldsymbol{\phi}_k(z)$ are generally used as a tool for identifying coherent structures in random processes and, in the present case, they may represent typical patterns of the fluctuating part of the FPUL.

Let $\varphi_k(z)$ be the angle in the *x*-*y* plane of the vector $\boldsymbol{\phi}_k(z)$ with respect to the *x*-axis. Figure 3 shows the angle $\varphi_1(z)$ evaluated at each instrumented level of the model and for different number of samples. It suggests that the most powerful coherent component of the FPUL acts in a plane oriented in somehow (by the angle φ_1) between the axes *x* and *y*. This is clearly strange for a process that is expected to be symmetric in some way. In the full paper it will be shown that this behaviour is not necessarily related to physical causes, but may be due to the sensitivity of $\boldsymbol{\phi}_1$ to lacks of symmetry of the pressure field as it can be argued noting that the angles φ_1 have wide statistical fluctuations.

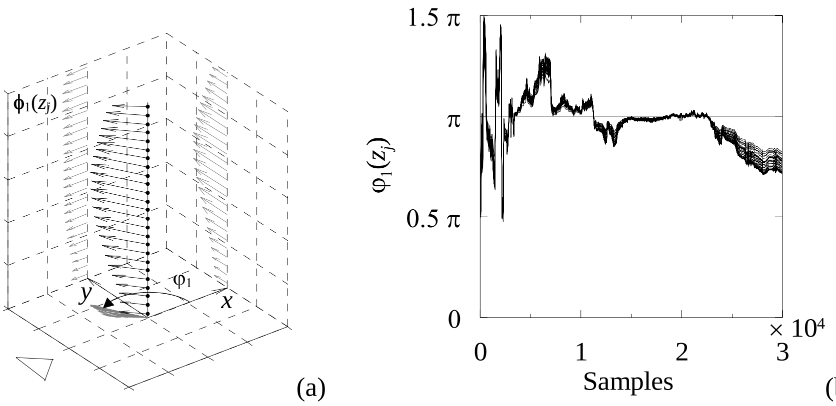

Figure 3. Angle in the *x*-*y* plane of the first covariance eigenfunction of the FPUL.

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REFERENCES

[1] H. Kikuchi, Y. Tamura, H. Ueda, K. Hibi. Dynamic wind pressure acting on a tall building model - Proper orthogonal decomposition. *Journal of Wind Engineering and Industrial Aerodynamics*, **69–71**, 631–646, 1997.

[2] G. Solari, L. Carassale, F. Tubino. Proper Orthogonal Decomposition in Wind Engineering: Part 1: A State-of-the-Art and Some Prospects. *Wind & Structures*, **10**(2), 177-208, 2007.