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POD-BASED IDENTIFICATION OF MODAL PARAMETERS FROM WIND-INDUCED STRUCTURAL RESPONSE

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1 INTRODUCTION

Conventional modal identification techniques extract modal parameters based on the information of both input loadings and output responses, which are called the input-output measurement approach [1]. The data for this approach can be obtained by exciting a structure with a shaker or by imparting an impact load on the structure. However, it is not easy or practical to vibrate a real structure to extract modal parameters, especially when the structure is large in size. To overcome this problem, the output-only measurement approach has been studied to identify modal parameters based on dynamic responses caused by natural external loads.

In this study, the Proper Orthogonal Decomposition (POD) technique is applied to evaluate the dynamic characteristics of a slender chimney structure based on the output data only, wind-induced dynamic responses. From the POD modes, the modal properties, such as natural frequencies, mode shapes, and modal damping ratios, are identified.

2 BACKGROUND OF POD ANALYSIS

In case of random displacement field, the POD modal vector can be obtained by solving the following eigenvalue problem:

$$[R]\{\phi\} = \lambda\{\phi\} \tag{1}$$

where $[R]$ is the space correlation matrix of displacement, and $\{\phi\}$ and λ are the POD mode shape vector and the corresponding eigenvalue, respectively. The POD modal vector $\{\phi\}$ satisfies the following orthogonality condition:

$$\{\phi\}_i^T \{\phi\}_j = \delta_{ij} \tag{2}$$

where δ_{ij} is the Kronecker delta. In the normal modal analysis, the mode shape vector is normalized as follows:

$$\{\psi\}_i^T [m] \{\psi\}_j = \delta_{ij} \tag{3}$$

where $\psi(z)$ is the mode shape vector and $[m]$ is mass matrix. By comparing with Eq. (2) and Eq. (3), the following relationship can be held [2]:

$$\{\psi\} = [m]^{-1/2} \{\phi\} \tag{4}$$

This implies that the normalized mode shape vector can be obtained by the POD mode shape vector if information of mass is available.

The displacement $u(z,t)$ can be reconstructed using the POD mode shape vector as follows:

$$u(z,t) = \sum_{i=1}^{n} a_i(t)\phi_i(z) \tag{5}$$

where $a(t)$ is the POD modal coordinate. The displacement $u(z,t)$ can be also obtained by the normal mode superposition method as follows:

$$u(z,t) = \sum_{i=1}^{n} q_i(t)\psi_i(z) \tag{6}$$

where $q(t)$ is the normal modal coordinate. It can be observed in Eqs. (5) and (6) that the normal modal coordinates can be obtained from the POD modal coordinates.

3 SIMULATION OF STRUCTURAL VIBRATION

3.1 Idealization of structural properties

The dynamic displacement responses obtained by numerical simulation were used to identify modal parameters of a structure using the POD method. The analysis model is a 45cm-thick cantilever-type chimney with exterior dimensions of $8.5m \times 8.5m \times 100m$ made of normal reinforced concrete. The structure is idealized as a 10 degrees of freedom. To induce the maximum displacement by resonance, the fundamental natural frequency of the structure is set to be equal to the frequency of the across-wind load. The damping matrix is constructed using the Rayleigh damping method with predetermined first and the second modal damping ratios, which are assumed to be 2.00% and 2.78%, respectively.

3.2 Wind tunnel test

A 1/250 scaled chimney model ($3.4cm \times 3.4cm \times 40cm$) was used to obtained the fluctuating wind pressure coefficients in a boundary layer wind tunnel as shown in Fig. (1) [3]. In the pressure test model, 120 pressure taps were used, and 16,384 sampling points of data were measured for each pressure tap with the frequency of 400Hz. In this study, the wind load is obtained from the across-wind surfaces, and the design wind speed is assumed to be 30m/sec.

Upper figures of Fig. (2) show the time history and power spectrum of the wind load acting on the 10th degree of freedom (D.O.F), where it can be observed from the peak of the spectrum that vortex shedding occurs around the frequency of 0.55Hz.

3.3 Wind-induced structural vibration

Bottom figures of Fig. (2) show the time history and the power spectrum of the displacement response at the 10th D.O.F obtained from the normal mode superposition method using the wind load and the assumed structural properties. It can be observed in the spectrum that the peak response occurs at the natural frequency of 0.55 Hz, which coincides with the frequency of vortex shedding. This implies that the structure is in resonance with the wind.

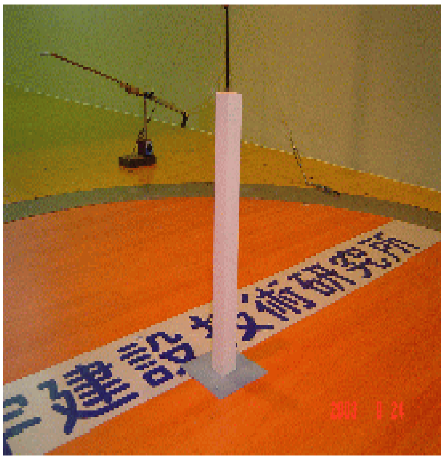

Figure 1: Setup for wind tunnel test

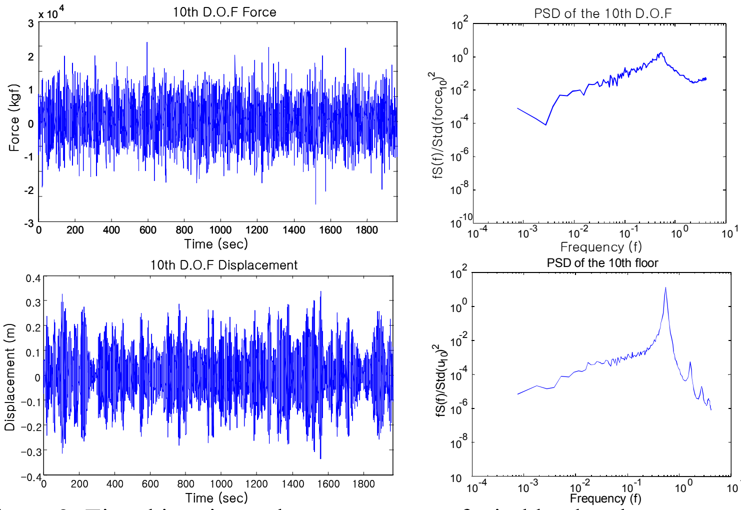

Figure 2: Time histories and power spectra of wind load and response

4 POD-BASED MODAL PARAMETERS

4.1 Mode shape vectors

Fig. (3) compares the first two POD mode shape vectors $\{\phi\}$ with those $\{\psi\}$ obtained by the normal modal analysis. It can be observed that the mode shape vectors obtained by the POD method correspond well with those obtained from the normal modal analysis.

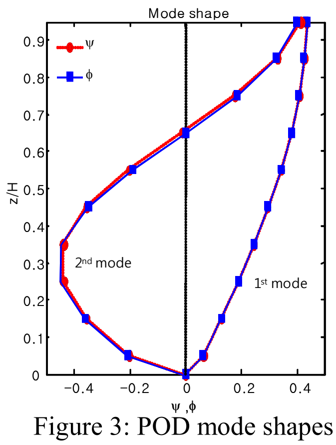

Figure 3: POD mode shapes

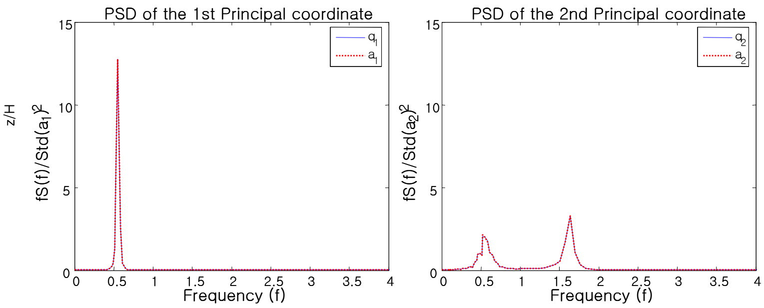

Figure 4: Power spectra of POD modal coordinates

4.2 Natural frequencies

Fig. (4) depicts the power spectra of the POD and normal modal coordinates (i.e., $a(t)$ and $q(t)$, respectively), where it can be seen that they correspond well with each other. It is possible to extract the natural frequencies from the maximum values (i.e., 0.55Hz and 1.64Hz for the 1st and 2nd modes, respectively) of the spectra in the POD modal coordinates. It can also observed that the secondary peak occurs at the forcing frequency of 0.55Hz in the 2nd mode.

4.3 Damping ratios

To estimate the modal damping ratios from the time histories of the POD modal coordinates, the random decrement (RD) method is applied [4]. Fig. (5) shows the extracted free vibration signatures for the 1^{st} and 2^{nd} POD modes by the RD method. The damping ratio of the POD mode is extracted from the mean value of the 15 damping ratios obtained from the 16 cycles of the free vibration signature using the logarithmic decrement method. Table 1 compares the damping ratios of the POD modes and the given damping ratios, where it can be observed that they match quite well within the error of 3.5 %.

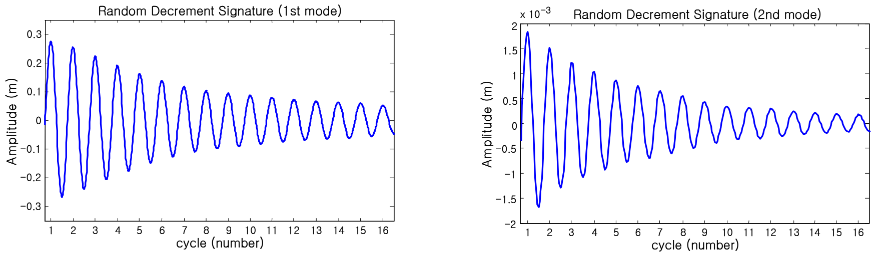

Figure 5: Free vibration signatures extracted by RD method

	Mode	Given value	POD value	Error (%)
Damping ratios (%)	1^{st}	2.00	1.93	3.50
	2^{nd}	2.78	2.83	1.80

Table 1: Comparison of modal damping ratios and those estimated by POD modes

5 CONCLUSIONS

In this study, the Proper Orthogonal Decomposition (POD) method, which is a statistical analysis technique to find the dynamic characteristics of a random data set, is applied to identify modal parameters of a tall chimney structure. The analysis results show that by using the POD technique the modal parameters, such as natural frequencies, mode shape vectors, and modal damping ratios, can be identified quite accurately based only on the dynamic responses of the structure. This implies that the POD method can be used as an output mode identifier which can complement the shortcomings of conventional input-output mode identifiers.

REFERENCES

[1] V. A. Herman. Structural Dynamics Modeling Using Modal Analysis. IEEE Instrumentation and Measurement Technology Conference, Budapest, Hungary, May 21−23, 2001.

[2] B. F. Feeny and Y. Liang. Interpreting proper orthogonal mode of randomly excited vibration systems. *Journal of Sound and Vibration*, 165, 953−966, 2003.

[3] J. Kim, D. Kim, H. Kim, and S. Kim. Estimation of Wind Load and Wind-Induced Responses on a Slender Structure Using Wind-Tunnel Test. Proceeding of the Korean Institute of Steel Structures, Seoul, Korea, June 5, 2004.

[4] C. J. Ku, J. E. Cermak, and L. S. Chou. Biased modal estimates from random decrement signatures of forced acceleration responses. *Journal of Structural Engineering*, ASCE, 133 (8), 1180-1185, 2007.