

## CHAOTIC VIBRATIONS IN A BUCKLED BEAM INDUCED BY A GALLOPING PHENOMENON

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**Keywords:** Aeroelasticity, Transverse galloping, Chaotic behavior

*ABSTRACT: Chaos theory has spectacularly evolved since the pioneering work by E. Lorenz on chaotic motion in a simple, deterministic system. Since then, the chaotic behavior of many other deterministic, low-dimensional systems in a large variety of fields has been developed. In the particular field of aeroelasticity of aircraft structures several reports of chaos have been documented. However, we are unaware of any report of chaotic systems of civil (non-aeronautical) use induced by an aeroelastic phenomenon. In this paper a well defined civil, aeroelastic system, susceptible to exhibit chaotic behavior is presented. The system consists of a buckled beam from which a second beam is suspended. This last beam (hereafter, galloping beam) has a square cross-section and can undergo transverse galloping. The system is subjected to an uniform wind flow and, as it will be shown in the paper, for wind velocities larger than a threshold value, the galloping beam begins to oscillate and induces, for a determined set of parameter values, a chaotic motion in the buckled beam.*

### *Mathematical model*

Let us consider a square cross-section beam (susceptible to transverse galloping) suspended from a buckled beam (it is not difficult to imagine a practical situation where, for example, the origin of the compression load could be due to thermal effects) under the action of an airstream with uniform velocity. Both beams are restricted to oscillate perpendicular to the airstream direction (Fig. 1). For the sake of simplicity, we assume that the vibrations of the system are mainly dominated by the lowest mode according to each degree of freedom. Thus, the system is analysed using a simplified model with two degrees of freedom that represents the balance of inertia, damping, restoring and aerodynamic forces. Quasi-steady conditions are assumed to model aerodynamic forces where we retain only the first nonlinear aerodynamic term. Furthermore, a simplified model of a buckled beam has been used [1]. In a non-dimensional form, the dynamics of the system is represented by the following non-linear differential system:

$$\ddot{\eta}_1 + c_{11}\dot{\eta}_1 - c_{12}(1 + c_{13})\eta_1 + c_{14}\eta_1^3 + c_{12}c_{13}\eta_2 = 0, \quad (1a)$$

$$\ddot{\eta}_2 + c_{15}\dot{\eta}_2 + c_{16}(1 + c_{17})\eta_2 - c_{16}c_{17}\eta_1 = c_{18}(U_r a_1 \dot{\eta}_2 + \frac{a_3}{U_r} \dot{\eta}_2^3), \quad (1b)$$

where,

$$\eta = y/D; \quad \tau = \omega_2 t; \quad c_{11} = \frac{c_1}{m_1 \omega_2} - \frac{\rho U^2 D a}{2 m_1 \omega_2}; \quad c_{12} = -\frac{k_1}{m_1 \omega_2^2}; \quad c_{13} = \frac{k_{12}}{k_1}; \quad c_{14} = \frac{k_{13} D^2}{m_1 \omega_2^2};$$

$$c_{15} = \frac{c_2}{m_2 \omega_2}; \quad c_{16} = \frac{k_2}{m_2 \omega_2^2} = 1; \quad c_{17} = k_{12}/k_2; \quad c_{18} = \frac{\rho D^2}{2 m_2}; \quad U_r = U/\omega_2 D; \quad (\dot{\cdot}) = d/d\tau.$$

and  $a_1, a_3$  are, respectively, the first and third order coefficients of aeroelastic force ( $a_1 = 2.7$  and  $a_3 = -168$  for a square section [2]).

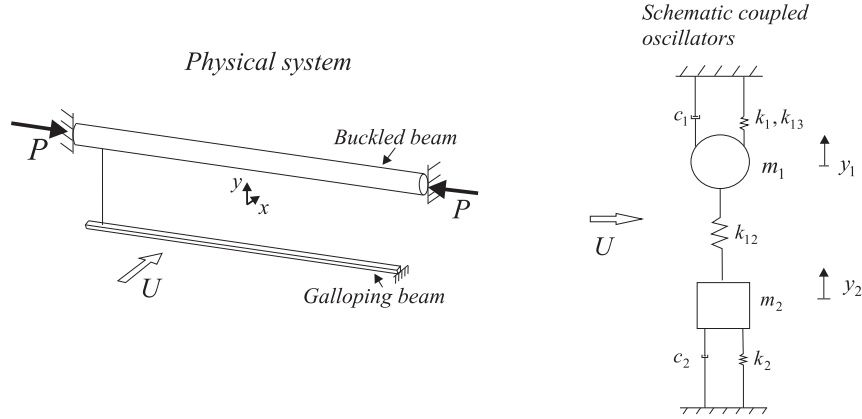


Figure 1: Graphical representation of the physical system and schematic model as a coupled oscillator.

The set (1) consists of two coupled non-linear and ordinary autonomous differential equations. Neither the system properties nor the forces action on the beams (we do not consider turbulence in the airstream) vary randomly with time and, consequently, the system is fully deterministic. If the nonlinearities appearing in the equations are strong, then a scenario of deterministic chaotic motion could appear as a consequence of the parameter values in the system.

Direct inspection of equations (1) does not lead to an easy, qualitative prediction on whether the system can turn chaotic. However, for values of the coupling parameter  $c_{17}$  small compared with the stiffness of the galloping beam,  $c_{16}$ , equations 1a and 1b decouples. Under this assumption,  $\eta_2$  in Eq. 1a becomes a forcing term with a known time dependence. This Eq. 1a becomes the extensively studied Duffing-Holmes equation that exhibits, as it is well known, a chaotic behavior within a certain parameter range.

### *Numerical results, bifurcation diagram and chaotic behavior*

In equations 1a and 1b there are several parameters characterizing the dynamic behavior. The high dimension of space parameter leads to the practical impossibility of complete parameter study. Thus, the study must restrict to a certain set of the most important physical variables in the problem. The most interesting one is to keep the system characteristics (e.g, mass, structural damping and stiffness) fixed and take the external actions (velocity of airstream and compressive axial load) as control parameters. To make the analysis as transparent as possible is analysed first the dynamic of the system with velocity of airstream as the only control parameter. Finally, a study with the two control parameters is presented.

### *Bifurcation diagram with $U_r$ as control parameter*

The system under study is in a slowly evolving environment, so its coefficient  $U_r$  experiments a gradual change. In nonlinear systems, behavior can change very suddenly and, depending of

the velocity of the wind, the buckled beam can be found in a state of static equilibrium, periodic oscillations around the equilibrium state or chaotic oscillations. Based on the precedent discussion, the original system (equations 1a and 1b) has been integrated with fixed values of the parameters. The parameters taken are  $c_{11} = 0.3^1$ ,  $c_{12} = 1.5$ ,  $c_{13} = -2/3$ ,  $c_{14} = 0.5$ ,  $c_{15} = 0.01$ ,  $c_{16} = 1$ ,  $c_{17} = 0.001$ ,  $c_{18} = 0.001$  and the wind parameter,  $U_r$ , is taken as the control parameter and bifurcation diagram has been computed in order to achieve the possible behaviors as the reduced velocity varies. In figure 2 the result obtained with a standard numerical Runge-Kutta algorithm is plotted. Four distinct regions according to the value of reduced velocity can be discern: (i)  $U_r < 3.7$  the instability of galloping has not begun yet and the buckled beam remains in the static equilibrium position, (ii)  $3.7 < U_r < 4.4$  the galloping beam is oscillating and induces a periodic vibration around the static equilibrium position in the buckled beam, (iii)  $4.4 < U_r < 6.6$  the oscillations in the buckled beam turn erratic and chaotic and (iv)  $U_r > 6.6$  vibrations in the buckled beam recover a periodic characteristic.

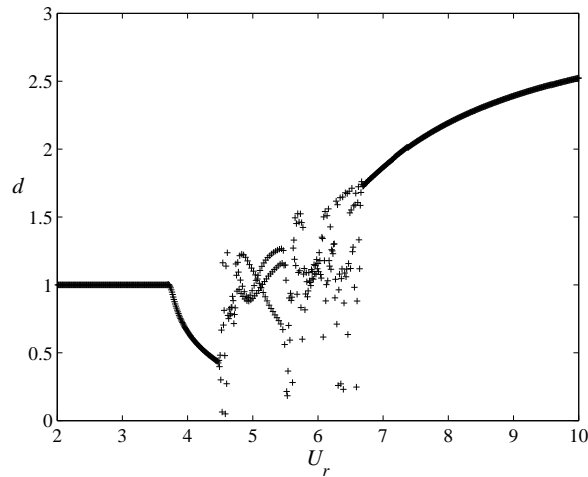


Figure 2: Bifurcation diagram for system with  $U_r$  as a control parameter and  $d = (\eta_1^2 + \dot{\eta}_1^2)^{1/2}$

From figure 2 it is clear that region (iii) can be considered as chaotic, although for values of reduced velocity between 4.8 and 5.4 the motion is not totally chaotic (numerical simulations in this region shows periods of chaos coexisting with periods with periodic motion). In figure 3 time series generated by numerical simulation after a considerable number of transient cycles and the corresponding phase projection for  $U_r = 6$  are shown. The top part of this figure shows the response of the buckled beam and the lower part the response of the galloping beam. From the figure can be observed that the buckled beam is oscillating in a somewhat erratic manner about, and between, the two static equilibrium positions, located at  $\eta_1 = \pm 1$ .

#### *Analysis of chaotic regions with the wind velocity, $U_r$ , and the compressive load, $c_{12}$ , as control parameters*

To investigate the possibility of chaotic motion a numerical study for the parameter of compressive load ranging from 0 to 5 and  $U_r$  varying from 0 to 12 has been carried out. Principal lower

<sup>1</sup>We consider that this parameter is dominated by structural damping and aerodynamic damping is a second order effect.

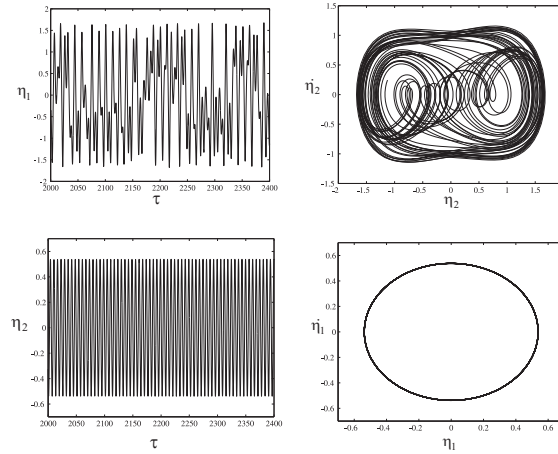


Figure 3: Numerical time series describing the position of the buckled beam,  $\eta_1$ , and the galloping beam,  $\eta_2$  for  $U_r = 6$ .

and upper  $U_r$  chaos boundaries were found for a discrete series of  $c_{12}$ . All results shown are for simulations started from the initial values of  $\eta_1 = 1, \dot{\eta}_1 = 0, \eta_2 = 0.03, \dot{\eta}_2 = 0, t = 0$ . From the figure can be observed that for no airflow or for no compressive load chaos does not occur. However, with both compressive load and airflow chaotic motion may occur as determined from numerical simulations. The higher the applied compressive load the greater wind velocity is necessary to the occurrence of chaos, and this takes place in a narrower range of reduced velocity. By contrast, when the compressive load is negative but moderate chaos would occur more easily.

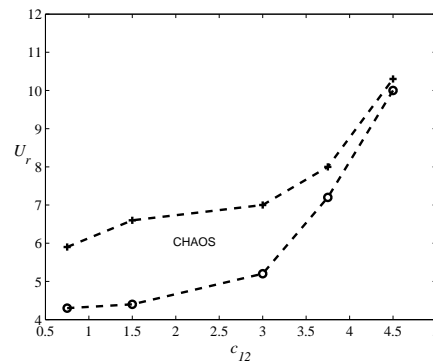


Figure 4: Sketch of chaotic region in the  $(c_{12}, U_r)$  parameter space.

## REFERENCES

- [1] Tseng, W. Y. & Dugundji, J. Nonlinear vibrations of a buckled beam under harmonic vibrations. *Transactions of the American Society of Mechanical Engineering Journal of Applied Mechanics*, 38:467-476, 1971.
- [2] Blevins, R. D. *Flow-Induced Vibration*, Krieger publishing company, New york, 1990.