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A COMPRESSIBLE CFD METHOD FOR FLOW WITH SOUND FROM VERY LOW MACH NUMBER TO SUPERSONIC

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Abstract

A compressible CFD method that can compute from very low Mach number with sound propagation to hypersonic flow accurately is presented. This method is based on MUSCL approach of third order spatial accuracy with a new Riemann flux function. The key idea is to control numerical viscosity in low Mach number. The formulation of the scheme are led and then some numerical examples, such as, sound propagation, vortex, flow around cylinder at various Mach number, are shown.

1 INTRODUCTION

Aeroacoustics is one of the subjects of most interest for the bluff body aerodynamics. Fluid motion and sound propagation are both governed by compressible Navier-Stokes equation, thus it is the most natural way to use compressible CFD method. However, there are some problems to use them directly for the flow of very low Mach number. One of these problems is the stiffness due to the huge ratio of characteristic speeds, however, this can be overcome by using implicit time integration and/or preconditioning method at least partially. Even when these methods are used, the design of the explicit scheme is quite important because accuracy and robustness of them depend more on the explicit part in schemes. Thus we concentrate on improvement of explicit scheme, although varieties of time integration methods are combined.

The biggest problem for using compressible CFDs, among those MUSCL with Riemann flux has been widely used, to low Mach number flows is excessive numerical viscosity contained in them. Numerical viscosity in Riemann flux is proper amount for transonic to supersonic flow, but it too much for very low Mach number. The key idea is controlling numerical viscosity in Riemann flux in a simple and robust manner.

2 NUMERICAL SCHEME

2.1 MUSCL approach with AUSM type Riemann flux

Governing equation is compressible Navier-Stokes equation. As inviscid terms have deterministic importance for high Reynolds number flow, only Euler equation is presented. The equation is discretized by cell centered finite volume method using 3rd order MUSCL. In this approach, distributions of primitive variables are reconstructed in each cell and then fluxes are computed by Riemann flux. This procedure can be written in one dimension as,

$$\mathbf{U}_{i+1/2}{}^{+} = \mathbf{U}_i + \frac{1}{4}\{(1+\kappa)(\mathbf{U}_{i+1} - \mathbf{U}_i) + (1-\kappa)(\mathbf{U}_i - \mathbf{U}_{i-1})\} \tag{1}$$

$$\mathbf{U}_{i+1/2}{}^{-} = \mathbf{U}_{i+1} - \frac{1}{4}\{(1-\kappa)(\mathbf{U}_{i+2} - \mathbf{U}_{i+1}) + (1+\kappa)(\mathbf{U}_{i+1} - \mathbf{U}_i)\} \tag{2}$$

$$\tilde{\mathbf{F}} = \tilde{\mathbf{F}}(\mathbf{U}_{i+1/2}{}^{+}, \mathbf{U}_{i+1/2}{}^{-}) \tag{3}$$

where $\mathbf{U}, \tilde{\mathbf{F}}$ are vectors of primitive variables and Riemann flux function, and $\kappa = 1/3$ is used. Liou and Steffen[1] developed AUSM(Advection Upstream Splitting Method) as a simple, robust Riemann flux scheme. Flux of AUSM can be written as,

$$\tilde{\mathbf{F}} = \frac{m+|m|}{2}\mathbf{\Phi}^{+} + \frac{m-|m|}{2}\mathbf{\Phi}^{-} + \tilde{p}\mathbf{N} \tag{4}$$

$$\Phi = (1, u, v, w, h)^T, \ N = (0, x_n, y_n, z_n, 0)^T \tag{5}$$

$$V_n = ux_n + vy_n + wz_n, \ m = \rho V_n, \ h = \frac{e+p}{\rho}, \ p = (\gamma-1)(e - \frac{\rho}{2}(u^2+v^2+w^2)) \tag{6}$$

where $\rho, e, p, \gamma, m, u, v, w, x_n, y_n, z_n$ are density, total energy, pressure, specific heat ratio, mass flux, components of velocity and normal vector of cell face respectively.

Shima and Jounouchi[2] pointed out that the overshoot generated by original AUSM is cured by replacing the mass flux with that of Roe scheme and proposed SHUS(Simple High Resolution Upwind Scheme). SHUS is used as basic scheme in this paper.

2.2 Controlling numerical viscosity at low Mach number

A Riemann flux function is essentially a first order upwind scheme, thus it contains numerical viscosity proportional to local characteristic speed. We will show AUSM type schemes also have that common nature in their pressure term inherited from a flux vector splitting scheme. The pressure term is written as,

$$\tilde{p} = \beta_+ p^+ + \beta_- p^- \tag{7}$$

$$\beta_{\pm} = \begin{cases} \frac{1}{4}(2 \mp M^{\pm})(M^{\pm} \pm 1)^2, \text{ if } |M^{\pm}| < 1 \\ \max(0, \min(1, \frac{1 \pm M^{\pm}}{2})), \text{if } |M^{\pm}| \geq 1 \end{cases} \tag{8}$$

where M denote Mach number normal to cell face. This is rewritten generally as,

$$\tilde{p} = \beta_+ p^+ + \beta_- p^- = \frac{p^+ + p^-}{2} + \frac{\beta_+ - \beta_-}{2}(p^+ - p^-) + \left(\left(\beta_+ - \frac{1}{2}\right) + \left(\beta_- - \frac{1}{2}\right)\right)\frac{p^+ + p^-}{2} \tag{9}$$

In low Mach number flow, neglecting higher order term, the last term approaches to,

$$\left(\left(\beta_+ - \frac{1}{2}\right) + \left(\beta_- - \frac{1}{2}\right)\right)\frac{p^+ + p^-}{2} \approx \frac{3}{4\gamma}\frac{\rho^+ + \rho^-}{2}\bar{c}(V_n{}^{+} - V_n{}^{-}) \tag{10}$$

where $\bar{c}$ is an average sound speed. Thus this term behaves as numerical viscosity. Scaling the numerical viscosity to the proper amount can be achieved by multiplying the non-dimensional factor that has size of Mach number. The simplest form is written as,

$$p = \frac{p_+ + p_-}{2} + \frac{\beta_+ - \beta_-}{2}(p_+ - p_-) + \min(1, \max(\widehat{M}^+, \widehat{M}^-))\left(\left(\beta_+ - \frac{1}{2}\right) + \left(\beta_- - \frac{1}{2}\right)\right)\frac{p_+ + p_-}{2} \tag{11}$$

$$\widehat{M} = \frac{1}{c}\sqrt{\frac{u^2 + v^2 + w^2}{2}} \tag{12}$$

Note that this scheme, LSHUS(Low-dissipation SHUS), behaves exactly same as SHUS in supersonic flow, so it keeps robustness when proper limiter is used in MUSCL.

3 NUMERICAL RESULTS

3.1 D‘Alemdert's paradox

It is known as D‘Alemdert's paradox that the two dimensional object in inviscid subsonic flow has no aerodynamic drag, but drag as error is caused in numerical solution. The drag coefficients of a circular cylinder in inviscid flow at several Mach numbers are shown in Fig.(1). Excessive numerical dissipation in Roe scheme and AUSM generate huge error in very low Mach number. On the other hand that of LSHUS is much lower and almost constant.

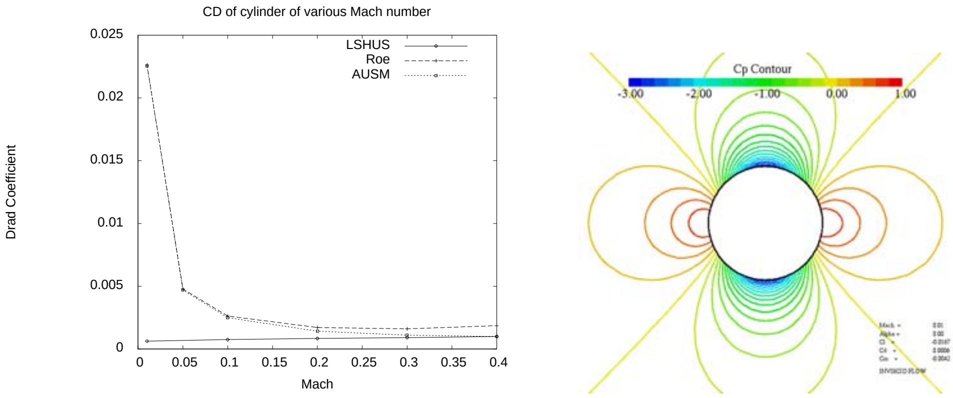

Figure 1: Drag of cylinder in inviscid flow. Cd vs. Mach(left) and Cp contour at Mach=0.01(right)

3.2 Rankine vortex

A static Rankine vortrex, of which radius is twice as large as grid size and peak Mach number is 0.01, is computed by several method. As shown in Fig.(2,3), results by Roe scheme and AUSM are smeared, on the other hand that of LSHUS almost keeps the initial shape.

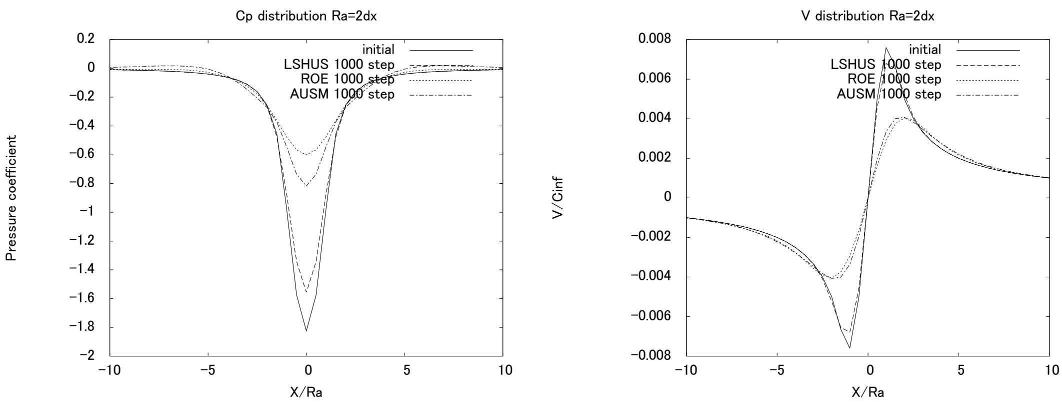

Figure 2: Cp(left) and velocity distribution(right) of a Rankine vortex after 1000 step.

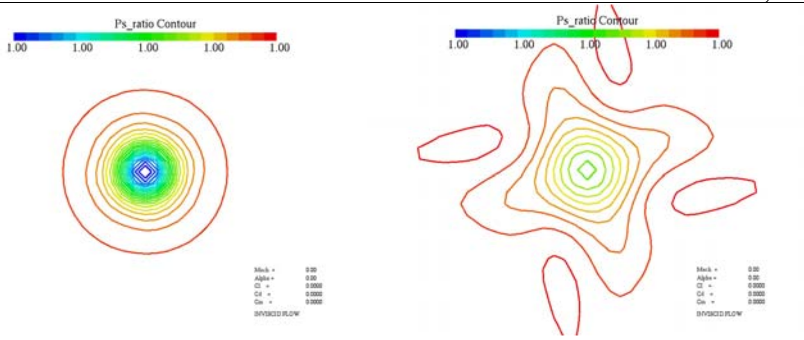

Figure 3: Pressure contour of a Rankine vortex after 1000 step. LSHUS(left) and Roe(right)

3.3 1D sound propagation

As the present scheme is based on compressible flow equation, sound propagation can be computed directly (Fig.(4)). LSHUS is less dissipative than Roe scheme.

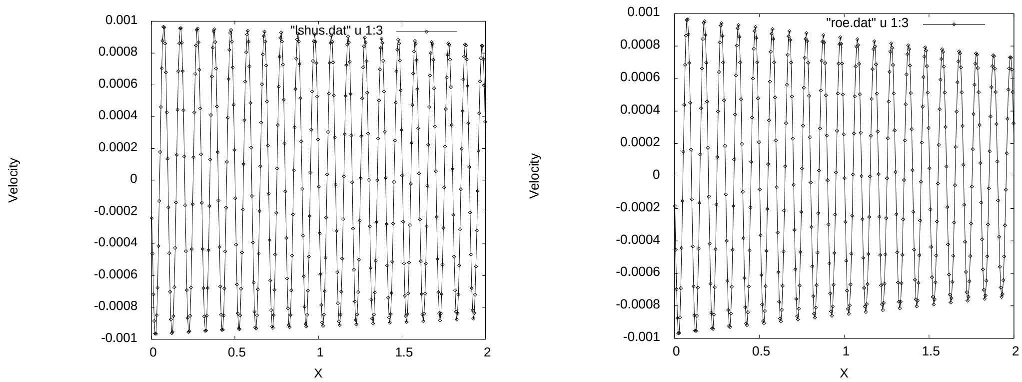

Figure 4: Velocity of one dimensional sound propagation. LSHUS(left) and Roe(right).

3.4 Sound wave generated by Karman vortices

As shown before, LSHUS has better accuracy in aerodynamics and sound propagation analysis, thus it is expected to have better accuracy in aeroacoustic problem. LSHUS captures fine vortices more sharply and higher frequency pressure fluctuation.(Fig.(5))

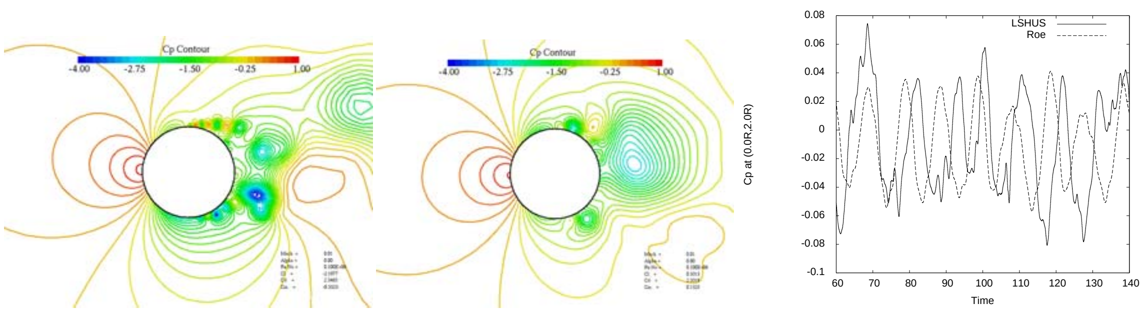

Figure 5: Cp contour by LSHUS(left), Roe(center) and Cp history at (0R,2R). M=0.01.

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