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EXTENDED LATTICE BOLTZMANN EQUATION FOR SIMULATION OF FLOWS AROUND BLUFF BODIES IN HIGH REYNOLDS NUMBER

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1 INTRODUCTION

Bluff bodies, say square cylinder and circular cylinder, have been widely used in practical engineering, such as bridge, high-rise building, and so forth. The prototype Reynolds number is always greater than 10^4 in turbulence state. In general, the direct numerical simulation (DNS), Reynolds averaged Navier-Stokes (RANS) and large eddy simulation (LES) are applied to expect the turbulence flow, which are on the basis of coarse grained model: Navier-Stokes (NS) equations [1]. However, the solution of these basic coarse-grained equations becomes mathematically difficult in the presence of turbulence. The simplified turbulence model used to modify the underlying coarse-grained equations still can not reliably reproduce many physical effects. The Lattice Boltzmann (LB) method is an effective numerical scheme for solving complex fluid dynamics problems, which was firstly proposed by McNamara and Zanetti [2], and it has gained rapid progress in development and application in the last two decades [3,4]. In solving turbulent flow problems by DNS based on LB method, one should resolve all these relevant excited degrees of freedom, and this is a virtually impossible task when Re>>1. It is thought that turbulence flow consists of two components: large-scale flows and small-scale fluctuations [1]. The contribution of the large-scale to momentum and energy transfer is computed exactly while the small-scale are modeled by turbulence model. Accordingly, the eddy viscous turbulence model is introduced to the LB equation to simulate the turbulence flows. In this paper, based on the theory of turbulence and molecule kinetics, the extended Lattice Boltzmann equation (ELBE) is derived to solve turbulent flows in high Reynolds number, in which sub-grid turbulence model is used to simulate vortex viscosity and turbulence relaxation time is introduced to modify the normal LBGK equation. Further more, the simulations of flows are performed successfully for square cylinder with Re=22,000 and circular cylinder with Reynolds number up to 140,000.

2 EXTENDED LATTICE BOLTZMANN EQUATION

It is thought that the flows are consisted of lots of particles in heat motion, and the behavior of many particle kinetic systems can be expressed as Boltzmann equation governing the single particle motions at molecular scales [4], namely,

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{x}}=\Omega(f) \tag{1}$$

where $f=f(\mathbf{x},\mathbf{v},t)$ is the particle distribution function; $\mathbf{v}$ is the particle velocity vector; $\mathbf{x}$ is the spatial position; t is the time variable. The left side of this equation represents the free streaming of molecules in space while the right side expresses intermolecular interactions or collisions. On the assumption that the details of the collision operator are immaterial, an effective collision operator represented by BGK (Bhatnagar, Gross, and Krook) expression is introduced as follows [3, 4]:

$$\frac{\partial f}{\partial t}+\mathbf{v}\cdot\nabla f=-\frac{1}{\lambda}(f-f^{(eq)}) \tag{2}$$

where λ is the relaxation time; $f^{(eq)}$ is the equilibrium distribution function. In order to solve f numerically, Eq. (2) is discretized in the velocity space with a finite set of velocity vectors $\{e_\alpha\}$ in the content of conservation laws, and then a completely discretized equation is derived with the time step Δt and space step $\Delta \mathbf{x}$, namely Lattice Boltzmann (LB) equation, as

$$f_\alpha(\mathbf{x}_i+\mathbf{e}_\alpha\Delta t,t+\Delta t)=f_\alpha(\mathbf{x}_i,t)-\frac{1}{\tau}\left[f_\alpha(\mathbf{x}_i,t)-f_\alpha^{(eq)}(\mathbf{x}_i,t)\right] \tag{3}$$

where f_α is the particle distribution function associated with the αth discrete velocity $\mathbf{e}_\alpha$; $\mathbf{x}_i$ is the location of spatial points; $\tau=\lambda/\Delta t$ is the dimensionless relaxation time.

The particle distribution function f and the equilibrium distribution function $f^{(eq)}$ describe the large-scale turbulent flows. In order to consider the unresolved small-scale fluctuations, an eddy viscous turbulence should be used in the LB equation, and the relaxation time τ is replaced by a total relaxation time τ_{total}. The extended LB equation is derived as

$$f_\alpha(\mathbf{x}+\mathbf{e}_\alpha\Delta t,t+\Delta t)=f_\alpha(\mathbf{x},t)-\frac{1}{\tau_{total}}(f_\alpha-f_\alpha^{(eq)}) \tag{4}$$

where $\tau_{total}=\tau_0+\tau_t$ is a total relaxation time depending on space and time; τ_0 and τ_t represent the contribution of the molecule viscosity υ_0 and the turbulence viscosity υ_t, respectively. τ_t includes the effect of subgrid-scale flows so that Eq. (4) keeps the large-scale eddies, but keeps out the small-scale eddies.

The remaining problem is to define the effective turbulent relaxation time τ_{total} describing the dynamics of turbulent fluctuations. Indeed, τ_{total} should depend on the variety of different turbulent physics, which includes the molecule relaxation time τ_0 and turbulent relaxation time τ_t.

$$\tau_{total}=\tau_0+\tau_t=3\frac{\Delta t}{\Delta x^2}(\upsilon_0+\upsilon_t)+\frac{1}{2} \tag{5}$$

The turbulence viscosity υ_t is governed by the Smagorinsky model [1] in this paper, as $\upsilon_t=(C_s\Delta)^2|\bar{S}|$, in which C_s is a Smagorinsky constant; Δ is the filter size; $|\bar{S}|=\sqrt{2\bar{S}_{ij}\bar{S}_{ij}}$ is the magnitude of the strain-rate tensor $\bar{S}_{ij}$. The quantity of $\bar{S}_{ij}$ can be calculated with resolved-scales non-equilibrium momentum tensor $\prod_{ij}=\sum_\alpha e_{\alpha i}e_{\alpha j}(f_\alpha-f_\alpha^{eq})$, as

$$\bar{S}_{ij}=-\frac{3}{2\bar{\rho}\tau_{total}\Delta t}\prod\nolimits_{ij}^{(1)}=-\frac{3}{2\rho\tau_{total}\Delta t}\sum_\alpha e_{\alpha i}e_{\alpha j}(f_\alpha-f_\alpha^{eq}) \tag{6}$$

There exists great difference between the behaviors of turbulence model in present ELBE and NS models e.g. RANS and LES.

3 SIMULATION OF FLOW AROUND SQUARE CYLINDER

A parallel LB algorithm that is suitable for muster-computers is developed, and the parallel computation code (LBFlow) is compiled using C++ computer language and MPI library. Further more, the simulation of flow around square cylinder is performed with *Re*=22000.The square cylinder is placed in a channel with uniform inflow shown as Fig. 1, and the coordinate system is set as follows: the *x*, *y* and *z* axes are the stream-wise, lateral and span-wise directions, respectively.

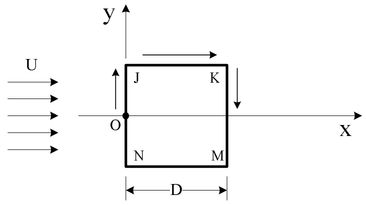
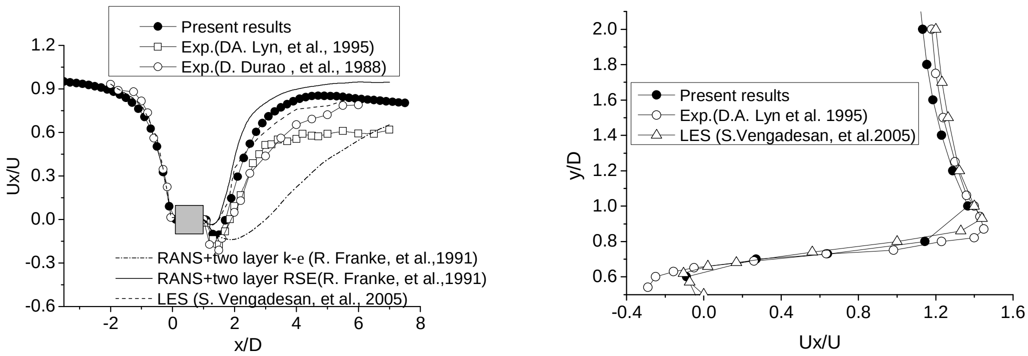


Fig.1 Geometry of the flow pass square cylinder

Table 1 lists the drag coefficient C_D and Strouhal number *St*. It shows that the present results agree well with that of experiments and LES method after detail comparison.

Method/source of result	C_D	St
Present results	2.084	0.140
Experiment (D.A. Lyn et al., 1995)	2.100	0.132
Experiment(D. Durão et al., 1988)	2.050~2.230	0.139
LES (S. Vengadesan, et al., 2005)	2.240	0.136

Table 1: Comparison of the drag coefficient and Strouhal number for square cylinder



(a) along the wake centerline (b) on the square cylinder side, x/D=0.5

Fig.2 Comparison of the stream-wise component of mean velocity

Fig.2 (a) and (b) illustrate the distribution of the time-averaged stream-wise velocity component along the wake centerline and along axis y with x/D=0.5, respectively. These quantities are compared with that of Lyn's experiment, Durao's experiment and other CFD methods including LES and RANS. From which, the present results have good agreement with Lyn's and Durão's experimental results though it is litter high while x/D>3; the mean stream-wise velocity along y direction at x/D=0.5 are in close proximity to that of experiment and LES.

4 SIMULATION OF FLOW AROUND CIRCULAR CYLINDER

Circular cylinder is also a typical bluff body which is often used to verify numerical model. The simulations of flows around circular cylinder are performed in the range of Reynolds number from 10 to 1.4×10^5.

Tab. 2 lists the drag coefficient C_D and Strouhal number St for circular cylinder with Re=3900, and 1.4×10^5. We can find that the present results agree well with that of the experiment and traditional NS-LES method after detail comparison.

Re	Method / Source of results	C_D	*St*
3900	Present results	1.1867	0.215
	Experiment (L.Ong, et al. 1996)	0.99±0.05	0.215±0.005
	LES (Kravchenko, et al. 2000)	1.00	0.203
	RANS + Standard ASM(H. Lübcke, et al., 2001)	0.89	0.200
	RANS + EASM (H. Lübcke, et al., 2001)	0.98	0.203
1.4×10^5	Presented results	1.2593	0.209
	Experiment（B. Cantwell, et al. 1983）	1.237	0.200
	LES（M. Breuer, 1999）	1.239	0.204
	RANS + EASM (H. Lübcke, et al., 2001)	1.16	0.220

Table 2: Comparison of the drag coefficient and Strouhal number for circular cylinder

5 CONCLUDING REMARKS

- The extended Lattice Boltzmann equation can been derived based on molecule kinetic theory and turbulence theory.
- The extended Lattice Boltzmann equation is suitable not only for steady flow at low Reynolds number but also for unsteady flow at high Reynolds number.
- In the cases of square cylinder and circular cylinder, the details of flow field vary with Reynolds numbers while the drag and vortexes shedding frequency are dependent on Reynolds numbers.

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