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POD-BASED IDENTIFICATION OF MODAL PARAMETERS FROM WIND-INDUCED STRUCTURAL RESPONSE

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Abstract. *In this paper, the Proper Orthogonal Decomposition (POD) method, which is a statistical analysis technique to find the modal characteristics of a structure, is adapted to identify the modal parameters of a tall chimney structure. As an applied load, a time history of wind force is obtained from a wind tunnel test of a scaled model. The POD method is applied on the wind-induced structural responses to predict the modal parameters of the structure. The analysis results show that the modal parameters including natural frequencies, mode shape vectors, modal damping ratios, and kinetic energy of the structure can be estimated accurately using the POD method. Based on the analysis results, it is concluded that the POD method can be applied to identify modal parameters of a structure accurately using the wind-induced responses.*

1 INTRODUCTION

The identification techniques for modal parameters, such as natural frequencies, mode shapes, and modal damping ratios, are frequently used for design and maintenance of structures.

Conventional modal identification techniques extract such parameters based on the information of both input loadings and output responses, which are called the input-output measurement approach, Ref. [1]. The data for input and output can be obtained by exciting a structure with a shaker or by imparting an impact load on the structure. However, it is not easy or practical to vibrate a real structure to extract modal parameters, especially when the structure is large in size. To overcome this problem, the output-only measurement approach has been studied to identify modal parameters based on dynamic responses caused by natural external loads such as wind or earthquake, Ref. [2].

The Proper Orthogonal Decomposition (POD) is a statistical analysis technique that has been applied in various engineering fields, Ref. [3] and Ref. [4]. In structural engineering field, the method was applied to investigate the dynamic characteristics of wind loads acting on structures, Ref. [5], Ref. [6], and Ref. [7]. Recently the POD method was applied to extract vibration modes of a simple beam specimen subjected to free vibration, Ref. [8] and Ref. [9].

In this study, the Proper Orthogonal Decomposition (POD) technique is applied to evaluate the dynamic characteristics of a slender chimney structure based on the output data only, wind-induced dynamic responses. From the POD modes, the modal properties, such as natural frequencies, mode shapes, and modal damping ratios, are identified.

2 BACKGROUND OF POD ANALYSIS

In case of random displacement field, the POD modal vector can be obtained by solving the following eigenvalue problem:

$$[R]\{\phi\} = \lambda\{\phi\} \tag{1}$$

where $[R]$ is the space correlation matrix of displacement, and $\{\phi\}$ and λ are the POD mode shape vector and the corresponding eigenvalue, respectively. The POD modal vector $\{\phi\}$ satisfies the following orthogonality condition:

$$\{\phi\}_i^{\,T}\{\phi\}_j = \delta_{ij} \tag{2}$$

where δ_{ij} is the Kronecker delta. In the normal modal analysis, the mode shape vector is normalized as follows:

$$\{\psi\}_i^{\,T}[m]\{\psi\}_j = \delta_{ij} \tag{3}$$

where $\psi(z)$ is the mode shape vector and $[m]$ is mass matrix. By comparing with Eq. (2) and Eq. (3), the following relationship can be held, Ref. [9]:

$$\{\psi\} = [m]^{-1/2}\{\phi\} \tag{4}$$

This implies that the normalized mode shape vector can be obtained by the POD mode shape vector if information of mass is available.

The displacement $u(z,t)$ can be reconstructed using the POD mode shape vector as follows:

$$u(z,t) = \sum_{i=1}^{n} a_i(t)\phi_i(z) \tag{5}$$

where $a(t)$ is the POD modal coordinate. The displacement $u(z,t)$ can be also obtained by the normal mode superposition method as follows:

$$u(z,t) = \sum_{i=1}^{n} q_i(t)\psi_i(z) \tag{6}$$

where $q(t)$ is the normal modal coordinate. It can be observed in Eqs. (5) and (6) that the normal modal coordinate can be obtained from the POD modal coordinate.

3 SIMULATION OF STRUCTURAL VIBRATION

3.1 Idealization of structural properties

The dynamic displacement responses obtained by numerical simulation were used to obtain the POD based modal parameters of a structure. The analysis model is a 45cm-thick cantilever-type chimney with exterior dimensions of $8.5m \times 8.5m \times 100m$ made of normal reinforced concrete. The structure is idealized as a 10 degrees of freedom system as shown in Fig. (1), and the mass, stiffness, and the damping matrices are presented in Table 1.

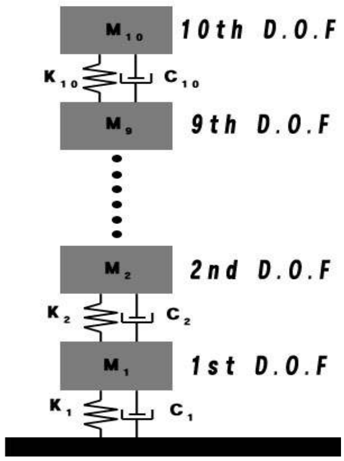

Figure 1. A 10 - DOF system

Table 1. System matrices of the model structure

Mass ($10^3 kgf \circ \sec^2/m$)					Stiffness ($10^7 kgf/m$)					Damping ($10^5 kgf \circ \sec/m$)				
35.5	0	..	0	0	3.8	−1.9	..	0	0	1.8	−0.9	..	0	0
0	35.5	..	0	0	−1.9	3.8	..	0	0	−0.9	1.8	..	0	0
.	.	..	.	.	.	.	..	.	.	.	.	..	.	.
.	.	..	.	.	.	.	..	.	.	.	.	..	.	.
0	0	..	35.5	0	0	0	..	3.8	−1.9	0	0	..	1.8	−0.9
0	0	..	0	35.5	0	0	..	−1.9	1.9	0	0	..	−0.9	0.9

To induce the maximum displacement by resonance, the fundamental natural frequency of the structure is set to be equal to the frequency of the across-wind load. The damping matrix is constructed using the Rayleigh damping method with predetermined first and the second modal damping ratios as follows, Ref. [10]:

$$[c] = \alpha[m] + \beta[k] \tag{7}$$

where the constants α and β are obtained by using the first and the second natural frequencies (ω_1 and ω_2, respectively) and the first and the second modal damping ratios (ζ_1 and ζ_2, respectively) as follows:

$$\alpha = \frac{2\omega_1\omega_2(\zeta_1\omega_2 - \zeta_2\omega_1)}{(\omega_2{}^2 - \omega_1{}^2)}, \beta = \frac{2(\zeta_2\omega_2 - \zeta_1\omega_1)}{(\omega_2{}^2 - \omega_1{}^2)} \tag{8}$$

In this study the first and the second modal damping ratios are assumed to be 2.00% and 2.78%, respectively, and the constants α and β are obtained as 0.0046 and 0.0831, respectively.

3.2 Wind tunnel test

A 1/250 scaled chimney model ($3.4cm \times 3.4cm \times 40cm$) was used to obtained the fluctuating wind pressure coefficients in a boundary layer wind tunnel as shown in Fig. (2), Ref. [11]. In the pressure test model, 120 pressure taps were used, and 16,384 sampling points of data were measured for each pressure tap with the frequency of 400Hz. In this study, the wind load is obtained from the across-wind surfaces, and the design wind speed is assumed to be 30m/sec.

Upper figures of Fig. (3) show the time history and power spectrum density function (PSD) of the wind load acting on the 10th degree of freedom (D.O.F), where it can be observed from the peak of the spectrum that vortex shedding occurs around the frequency of 0.55Hz.

3.3 Wind-induced structural vibration

Bottom figures of Fig. (3) show the time history and power spectrum density function of the displacement response at the 10th D.O.F obtained from the normal mode superposition method using the wind load and the assumed structural properties. It can be observed in the spectrum that the peak response occurs at the natural frequency of 0.55Hz, which coincides with the frequency of vortex shedding. This implies that the structure is in resonance with the wind.

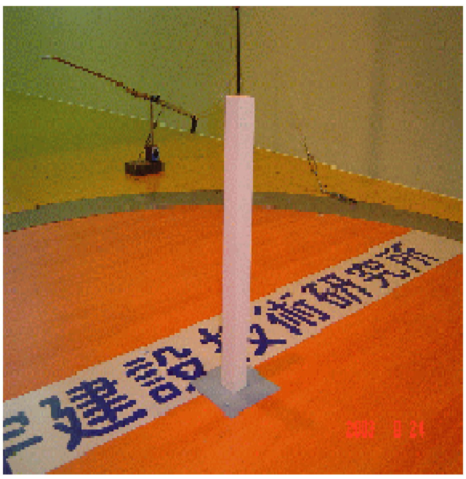

Figure. 2. Setup for wind tunnel test

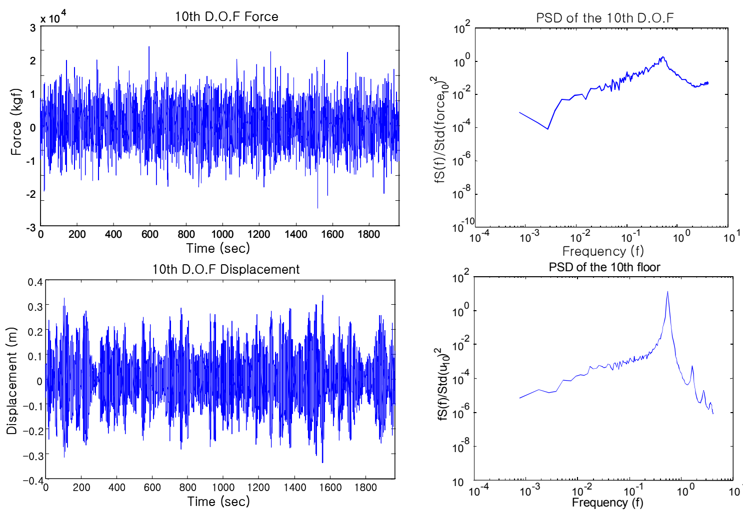

Figure. 3. Time histories and PSD of wind load and response

4 POD-BASED MODAL PARAMETERS

The POD technique is applied on the structural responses obtained from the numerical simulation and wind-tunnel test, and major modal parameters are indentified.

4.1 Modal kinetic energy

Table 2 shows the eigenvalues of the POD modes, which represent the modal kinetic energy of the structure. It can be noticed that most of the kinetic energy is contributed from the first mode of vibration, and there is a negligible contribution of the modes higher than the 3rd mode.

Mode	λ_i	$\frac{\lambda_i}{\sum \lambda_i} \times 100(\%)$
1st	864.62	99.98
2nd	0.15	0.02
3rd	0.01	0.00

Table 2. POD Energy distribution of POD modes

4.2 Mode shape vectors

Fig. (4) compares the first three normal mode shape vectors with those obtained by the POD method. It can be observed that the mode shape vectors obtained by the POD method corresponds well with those obtained from conventional modal analysis.

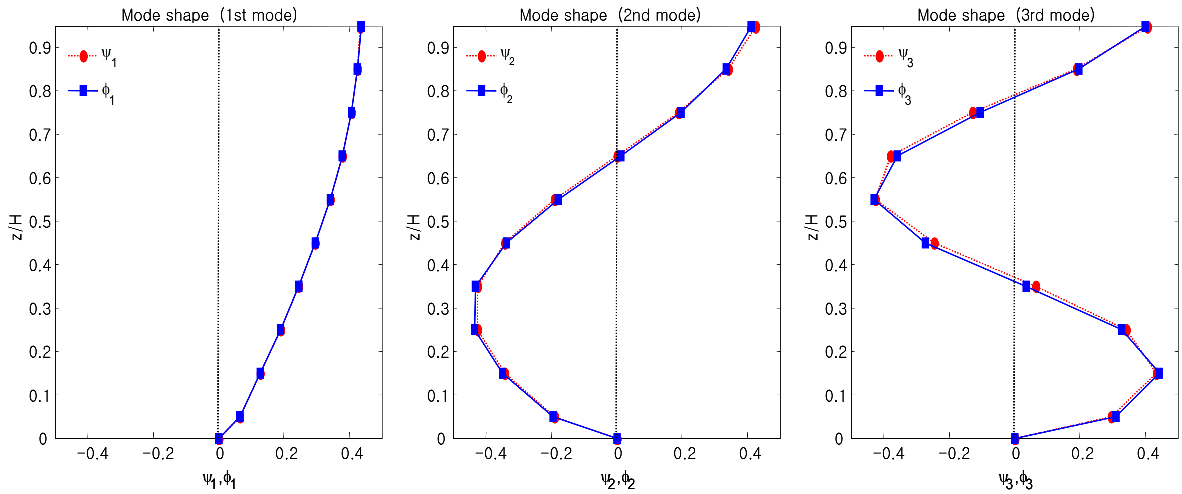

Figure. 4. Comparison of exact mode shapes and those estimated by POD modes

4.3 Natural frequencies

Fig. (5) depicts the power spectrum density functions of the POD and conventional modal coordinates, where it can be seen that they correspond well with each other.

It is possible to extract the natural frequencies from the maximum values of the power spectrum density functions in the POD modal coordinates, and Table 3 compares the natural frequencies obtained by the conventional method and the POD analysis, where it can be observed that the natural frequencies match quite well. It also can be observed that secondary peaks are also observed around the forcing frequency of 0.55Hz in the 2nd and the 3rd modes.

This implies that the forcing frequency as well as the natural frequencies can be identified from the POD modal coordinates.

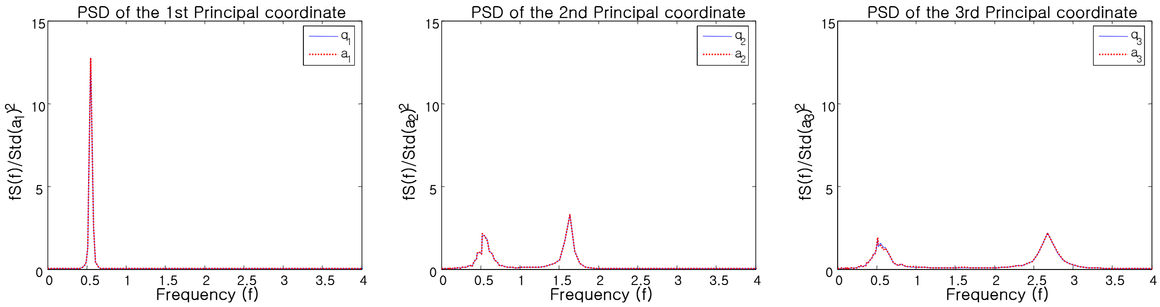

Figure. 5. Power spectra of POD modal coordinates

Table 3. Comparison of natural frequencies (normal modal analysis versus POD analysis)

Mode	Natural frequency (Normal modal analysis)	Natural frequency (POD analysis)	Error (%)
1st	0.55	0.55	0.00
2nd	1.64	1.64	0.00
3rd	2.69	2.68	0.37

4.4 Damping ratios

To estimate the modal damping ratios from the displacement time histories in the POD modal coordinates, the random decrement (RD) method, Ref. [12] and Ref. [13], is applied. The RD method divides random time history data into sub-samples that satisfy a given condition and superposes the samples to extract free vibration records. In this paper sub-samples are extracted starting from twice the RMS value of POD coordinates. As it can be observed in Fig. (6), the ensemble averaged values are obtained by superposing $+a(t)$ when the gradient of a sub-sample at starting point is positive and $-a(t)$ when the gradient is negative.

Fig. (7) shows the time histories of POD modal coordinates. 200 sub-samples are superposed using the RD method described in Fig. (6) to extract the free vibration records presented in Fig. (8). The damping ratio of the POD mode is extracted from the mean value of the 15 damping ratios obtained from the 16 cycles of free vibration shown in Fig. (8) using the logarithmic decrement method. Table 4 compares the damping ratios of the POD modes and the given damping ratios, where it can be observed that they match quite well within the error of 3.5 %.

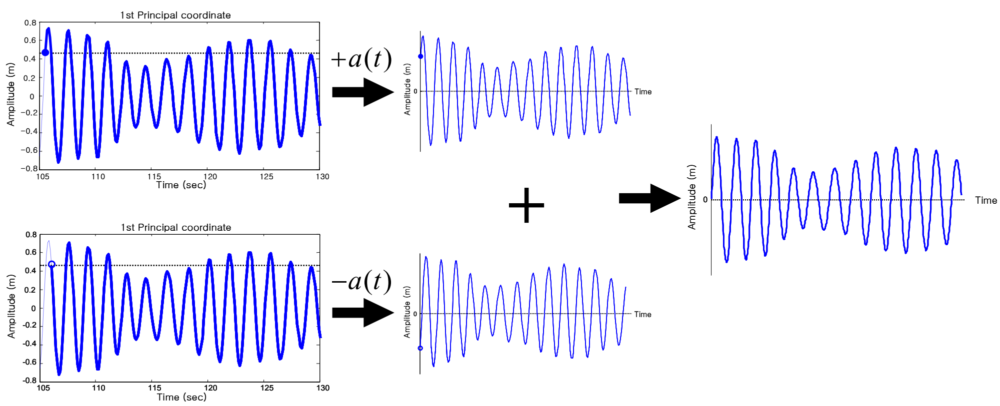

Figure. 6. Superposition of samples in random decrement method

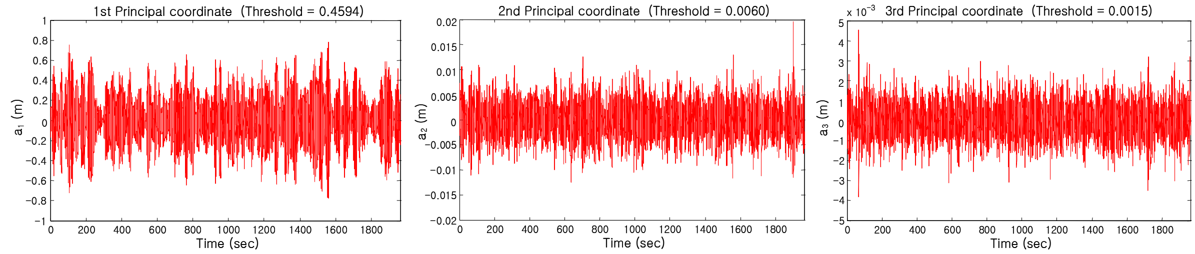

Figure. 7. Time histories of POD modal coordinates

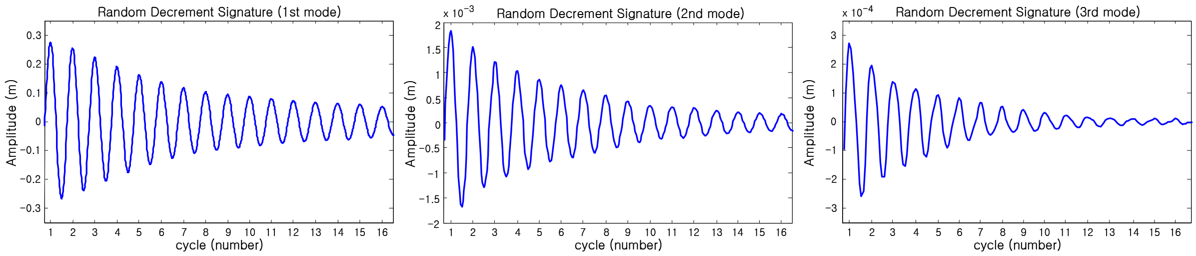

Figure. 8. Free vibration records extracted by RD method

	Mode	Given value	POD mode	Error (%)
Damping ratio (%)	1st	2.00	1.93	3.50
	2nd	2.78	2.83	1.80
	3rd	4.15	4.13	0.48

Table 4. Comparison of modal damping ratios and those estimated by POD modes

5 CONCLUSIONS

In this study, the Proper Orthogonal Decomposition (POD) method, which is a statistical analysis technique, is applied to find the dynamic characteristics of a chimney-like structure. The modal parameters such as natural frequencies, mode shape vectors, and the modal damping ratios were identified using wind-induced responses, and they were compared with exact solutions. The analysis results showed that by using the POD technique the modal parameters could be identified quite accurately based only on the dynamic responses of a structure. This implies that the POD method can be used as an output mode identifier which can complement the shortcomings of conventional input/output mode identifiers.

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REFERENCES

[1] B. Schwarz and M. Richardson. Experimental Modal Analysis. Proceedings of the CSI Reliability Week, Florida, USA, October 4−7, 1999.

[2] W. Ren and Z. Zong. Output-only modal parameter identification of civil engineering structures. *Structural Engineering and Mechanics*, **17**, 1−16, 2004.

[3] J. Lumley. *Stochastic Tools in Turbulence.* Academic Press, New York, USA, 1970.

[4] P. Holmes, J. Lumley, and G. Berkooz. *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*. Cambridge University Press, New York, USA, 1996.

[5] H. Ham. *Turbulence effects on wind-induced building pressure*. Ph.D. Dissertation, Colorado State University, USA, 1998.

[6] A. Katsumura, Y. Tamura, and O. Nakamura. Universal wind load distribution simultaneously reproducing largest load effects in all subject members on large-span cantilevered roof. *Journal of Wind Engineering and Industrial Aerodynamics*, **95**, 1145−1165, 2007.

[7] X. Chen and N. Zhou. Equivalent static wind loads on low-rise buildings based on full-scale pressure measurements. *Engineering Structures*, **29**, 2563−2575, 2007.

[8] S. Han and B. Feeny. Application of proper orthogonal decomposition to structural vibration analysis. *Mechanical Systems and Signal Processing*, **17**, 989−1001, 2003.

[9] B. Feeny and Y. Liang. Interpreting proper orthogonal mode of randomly excited vibration systems. *Journal of Sound and Vibration*, **165**, 953−966, 2003.

[10] A. Chopra. *Dynamics of structures: theory and applications to earthquake engineering*. Prentice-Hall, New Jersey, USA, 2001.

[11] J. Kim, D. Kim, H. Kim, and S. Kim. Estimation of Wind Load and Wind-Induced Responses on a Slender Structure using Wind-Tunnel Test. Proceeding of the Korean Institute of Steel Structures, Seoul, Korea, June 5, 2004.

[12] A. Jeary. The description and measurement of nonlinear damping in structures. *Journal of Wind Engineering and Industrial Aerodynamics*, **59**, 103−114, 1996.

[13] C. Ku, J. Cermak, and L. Chou. Biased modal estimates from random decrement signatures of forced acceleration responses. *Journal of Structural Engineering*, ASCE, **133**, 1180-1185, 2007.