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STRUCTURAL OPTIMIZATION OF THE FLUTTER PROBLEM IN SUSPENSION BRIDGES: AN ANALYTICAL FORMULATION

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Abstract. *It is well known that long span suspension bridges are wind prone structures. For decades guaranteeing the safe performance of bridges against flutter has been one of the main concerns of designers as that aeroelastic instability can be responsible for the bridge destruction. To date, the project of cable supported bridges has been essentially a trial and error procedure. Nevertheless, the maturity gained by the scientific community in the study of aeroelastic phenomena, the development of general optimum design theory and the improvements in the computer power make feasible the application of optimum design techniques in the project of suspension bridges considering the flutter performance as a behaviour constraint. In this work the analytical formulation of the optimization process of box girder suspension bridges considering flutter and kinematic constraints is reviewed. Attention is paid to the analytical evaluation of the flutter speed derivatives. Moreover a reference is introduced to the Messina Bridge application example presented by the authors in the CWE2006 conference.*

1 INTRODUCTION

Cable supported bridges play today a key role in ground transportation networks worldwide. Today's social and economical developments suggest that the project and construction of these challenging structures is going to be even more intense in the future. Moreover, technical and scientific skills of the bridge and wind engineering communities have reached an unprecedented level. Therefore, it is worthy to take some time to reflect on the design techniques applied in the bridge engineering realm and the specific field of wind effects on long span bridges.

To date, long span bridges have been projected using conventional design techniques. The conventional design method consists in a trial and error procedure that the designer, based upon his experience and expertise, carries out to bring the prototype into accordance with the set of imposed constraints. In Fig. (1) a flowchart containing the basic stages to follow is presented. However there is no guarantee for the final design, in spite of fulfilling the required specifications, to be the best of all the possible designs which satisfy the project requirements. In addition, along the design process a modified design can perform worse than a previous one, making the design process long, complex, tedious and unfeasible. Comprehensive treatises about general bridge design have focused exclusively on the classical design method [1], [2], [3]. A relevant exception is the book by Troitsky [4] which devotes some space to the optimal design of continuous composite steel bridge girders.

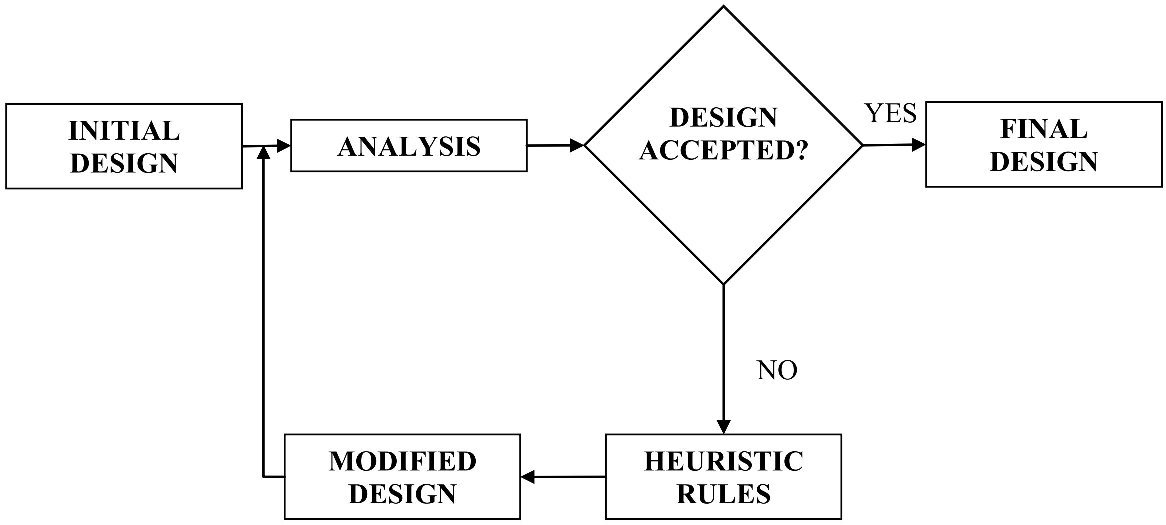

Figure 1: Conventional design flowchart.

The alternative approach is any of the non-conventional design techniques, that is, design based upon sensitivity analysis and optimum design. Comprehensive references can be found in technical literature [5], [6], [7], [8]. Practical applications in structural problems of the aforementioned non-conventional design techniques are not just an academic issue. These techniques have been successfully applied for decades in extremely competitive sectors such as car and aeronautic industries.

Sensitivity can be defined as a response derivative with regards to a design variable, that is, a structural property with a potential for change. This derivative can be understood as the expected change in the response when the considered design variable is perturbed. Therefore, the structural behaviour due to a change in a design variable can be anticipated. This circumstance allows the designer to follow a guided design process avoiding unfruitful design modifications. In Fig. (2) the scheme of a sensitivity analysis based design process is shown. Practical applications of sensitivity analysis to bridge engineering are scarce [9], [10].

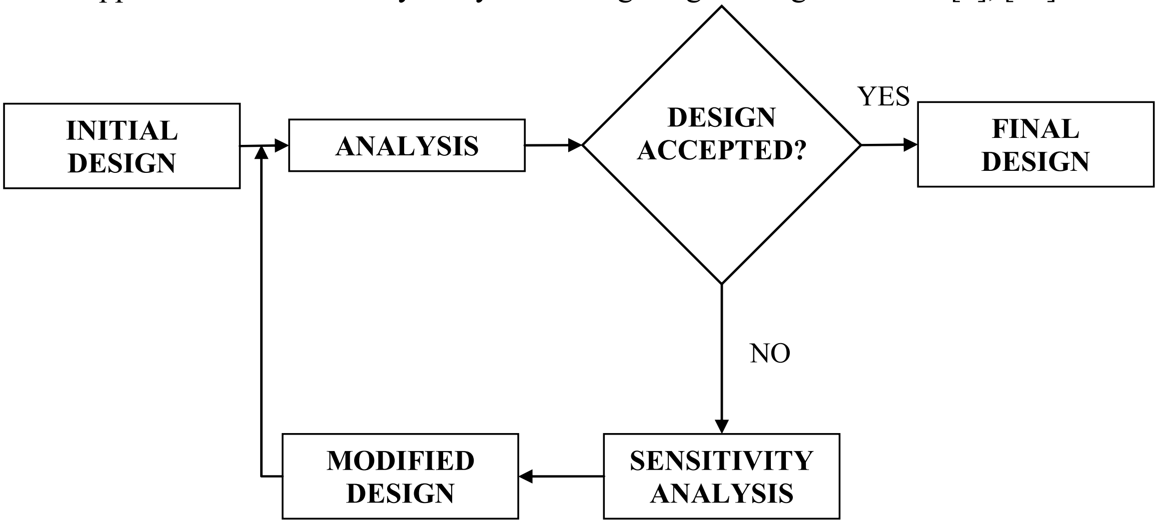

Figure 2: Sensitivity analysis based design flowchart.

A classical definition for optimum design is the one by Wilde [11] "the best feasible design according to a preselected quantitative measure of effectiveness". In optimal structural design a certain objective function, (structure weight in many cases) must be minimized or maximized by modifying the design variables while satisfying a set of behaviour and design constraints. Thanks to the use of computers the final optimum design can be accomplished by mathematical methods. This efficient and logical approach contrasts with the use of heuristic rules which characterizes the conventional design process. Several examples of structural optimization can be found in the literature [12], [13]. In Fig. (3) the conceptual scheme of an optimum design process is presented.

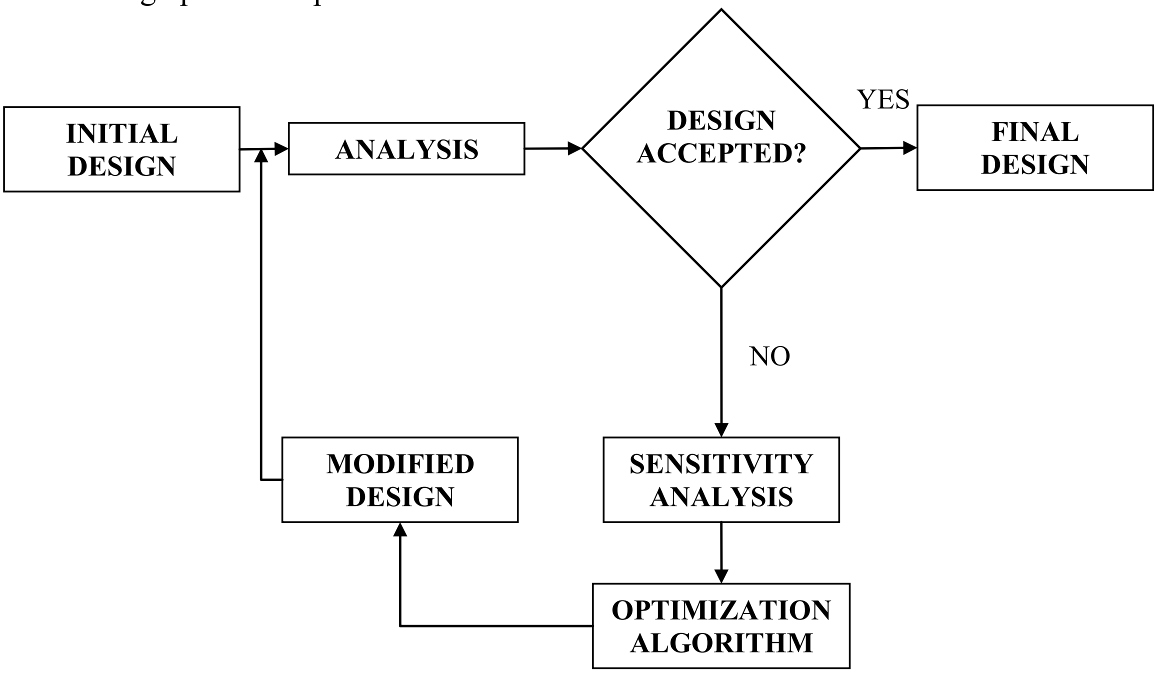

Figure 3: Optimum design flowchart.

It is well known that long span suspension bridges are wind prone structures. They can be affected by a number of aerodynamic and aeroelastic phenomena such as vortex induced vibrations, galloping or flutter, amongst others. For decades, guaranteeing the safe performance of bridges against flutter has been one the main concerns of designers as that aeroelastic instability can be responsible for the complete destruction of the structure.
Therefore, a lot of work has been devoted to obtain bridge decks whose cross-section performs feasibly from the aerodynamic point of view. A clear example has been the wind tunnel testing campaign of the Great Belt Bridge where 1:80 scale section model tests were conducted in smooth and turbulent flow with the aim of evaluating the flutter performance of a series of candidate section designs [14]. Moreover, altogether with its aerodynamic characteristics it is possible to modify the structural properties of the bridge deck during the design phase in order to improve the bridge performance against flutter. As a matter of fact, the Structural Mechanics Research Team of the University of La Corunna has been working for years in the application of non-conventional design techniques in cable supported bridges considering their flutter performance. Applications of sensitivity analysis can be found in several publications [15], [16], [17], [18]. The gained confidence in sensitivity analysis applied to the design of cable-supported bridges allows facing the aeroelastic optimum design of such challenging structures. Along the present document the optimum problem formulation will be commented.

2 FORMULATION OF THE OPTIMUM DESIGN PROBLEM OF SUSPENSION BRIDGES CONSIDERING AEROELASTIC AND KINEMATIC CONSTRAINTS

2.1 Optimum design problem formulation

Mathematically, the optimization problem can be written as

$$\min F(\mathbf{x}) \tag{1}$$

Which means that a certain objective function F, that depends upon a set of design variables $\mathbf{x} = \left(x_1, \ldots, x_n\right)$, being n the number of selected design variables, must be minimized, or maximized, if that is the case.

Additionally, a set of design and behaviour constraints must be accomplished:

$$g_j\left(\mathbf{x}\right) \leq 0 \quad j = 1, \ldots, l \tag{2}$$

and

$$x_{i\min} \leq x_i \leq x_{i\max} \tag{3}$$

Where l is the total number of chosen constraints; and x_i must be inside an interval of appropriated values.

In the present work, the optimization problem to be solved is to obtain the minimum weight bridge deck compatible with a stipulated minimum flutter critical wind speed and a maximum allowed deck vertical displacement. Design variables may change continuously inside a range between a lower limit and an unbounded upper limit. Therefore the optimum design problem to be solved is:

$$\min F = A(e_1, ..., e_n) \tag{4}$$

Subject to:

$$e_{\min} \leq e_i \leq e_{\max} \quad i = 1, ..., n \tag{5}$$

$$g_1: \ \frac{U_{f,cr}}{U_f} - 1 \leq 0 \tag{6}$$

$$g_2 : \frac{w_c}{w_{c,\max}} - 1 \le 0 \tag{7}$$

In the former expressions A is the area of the deck cross-section. The deck weight is directly related with the cross-section area; therefore, the minimum area deck corresponds with the minimum weight deck. When the suspension bridge deck is a box girder, then it is clear that the cross-section area is dependent upon the material thicknesses ($e_1,\ldots,e_n$). Material thicknesses must be included inside a certain range whose lower bound $e_{\min}$ is the minimum thickness due to manufacturing or constructive considerations. In practical applications the upper bound $e_{\max}$ can be neglected. In section 2.6 the relationship between a box cross-section area and material thicknesses will be surveyed in depth.

The behaviour constraints to be satisfied are the following: The flutter speed U_f must be equal or higher than a critical value $U_{f,cr}$ established in the project requirements. Additionally, the vertical displacement of the span centre w_c must be equal or lower than a maximum bound $w_{c,\max}$ given by the project requirements or national codes for a certain set of actions.

The optimization method to be implemented for solving the design problem is the modified feasible directions method [7]. This is a gradient based method, therefore, objective function and constraints gradients must be supplied to the optimization algorithm. In spite of their popularity, finite differences are strongly discouraged from being used in the evaluation of the aforementioned gradients due to step size uncertainty, risk of mode order change and the enormous amount of the required computer time to evaluate, amongst others, the derivatives of the flutter speed with regard to the design variables. The most efficient strategy developed to date for carrying out this task is the analytical evaluation of the derivatives. It must be borne in mind that the mentioned gradients must be evaluated repeatedly along the optimum design process which makes worthy the effort devoted to obtain the analytical expressions of the derivatives and their implementation in a computer code.

The correct optimum problem formulation and an adequate numerical strategy are key issues, nevertheless they are not enough in order to guarantee reaching the optimal solution. In the aeronautic realm one of the basic concerns of designing under flutter constraints is the discontinuous dependency of flutter upon structural stiffness. Thus, flutter constraint may not be a continuous function of the structural design variables due to the effect of “hump” modes [19], [20]. This aspect should be carefully checked whenever the aeroelastic optimum design of long span bridges is carried out.

2.2 Structural finite element modelling for aeroelastic optimum design

Suspension as well as cable-stayed bridges may be analyzed by means of three dimensional finite element models adopting beam elements, multiple references can be found in the literature as [21]. Thus, the first step in the optimum design process must be to work out a bridge three dimensional beam elements model bearing in mind that the great amount of repetitive calculations to be carried out entails the number of degrees of freedom to be moderate [22]. In Fig. 4 an image of a 3D finite element model of a suspension bridge is presented.

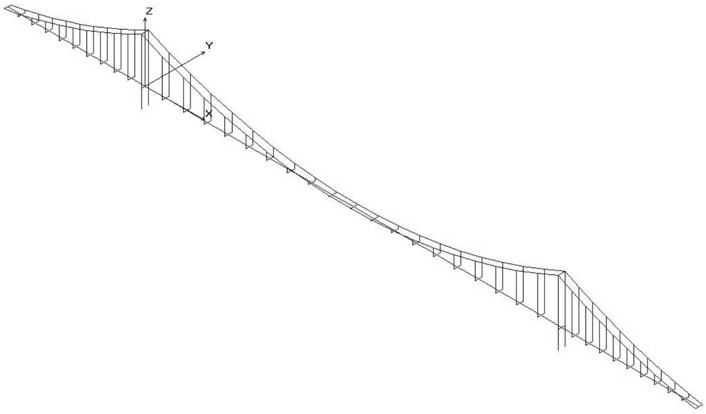

Figure 4: Finite element model of a suspension bridge.

For analysis purposes the deck mass per unit of length is expressed as

$$\underline{m}_x = \underline{m}_y = \underline{m}_z = \rho A + \underline{m}^{ns} \tag{8}$$

Where the symbol _ highlights that the parameter is expressed per unit of length, ρ is the deck material density, A is the deck cross-section area and $\underline{m}^{ns}$ is the non-structural mass per unit of length, which represents the mass contribution of bulkheads, stiffening members or the bearing surface.

Similarly, the deck polar moment of inertia $\underline{m}_{\varphi x}$ is evaluated as:

$$\underline{m}_{\varphi x} = \rho(I_y + I_z) + \underline{m}_{\varphi x}^{ne} \tag{9}$$

Where I_y is the deck cross-section vertical moment of inertia, I_z is the deck cross-section lateral moment of inertia and $\underline{m}_{\varphi x}^{ns}$ is the non-structural elements contribution to the polar moment of inertia per unit of length. Along the optimum design process non-structural contributions in (8) and (9) are considered to remain fixed.

In Fig. 3 the conceptual flowchart of the optimum design process has been presented when a gradient based optimization method is used. According with the optimization problem stated in Eqs. (4) to (7), two are the core tasks to be completed along the design process: performing the aeroelastic analysis, that is, evaluation of the flutter speed in the studied case, and obtaining the sensitivity analyses of the flutter velocity and the deck displacements with regard to the considered design variables. These aspects will be reviewed next.

2.3 Evaluation of the bridge flutter velocity

The hybrid method has been used in order to obtain the bridge flutter speed [23], [24]. This method is called hybrid because it comprises two different phases. In first place, an experimental one with the aim of obtaining the flutter derivatives and, once this has been completed, a computational phase begins which allows the calculation of the bridge flutter speed U_f. The main operations concerned with the calculation of the flutter speed are the following:

- Wind tunnel section model testing for obtaining up to 18 flutter derivatives (A_i^*, H_i^*, P_i^* $i = 1, \ldots, 6$) which are a function of the reduced frequency $K=B\omega/U$. Being B the deck width, ω the frequency of the response and U the oncoming wind speed.
- Calculation of the bridge natural frequencies and mode shapes using a three-dimensional finite element model. This is a mandatory step as modal analysis will be applied to obtain

the flutter speed. A key issue in suspension bridges is the stiffening effect due to tension in cables. Thus, the stiffness matrix in second order theory must be adopted [25].

$$\mathbf{K}_{\text{nonlin}} = \mathbf{K}_{\text{lin}} + \mathbf{K}_{\text{G}} \tag{10}$$

Where $\mathbf{K}_{\text{lin}}$ is the linear theory stiffness matrix and $\mathbf{K}_{\text{G}}$ is the geometrical stiffness matrix.

- Computational evaluation of the aeroelastic forces acting on the bridge deck.

$$\begin{aligned}\begin{pmatrix} D \\ L \\ M \end{pmatrix} &= \frac{1}{2}\rho U^2 \begin{pmatrix} K^2P_4^* & -K^2P_6^* & -BK^2P_3^* \\ -K^2H_6^* & K^2H_4^* & BK^2H_3^* \\ -BK^2A_6^* & BK^2A_4^* & B^2K^2A_3^* \end{pmatrix}\begin{pmatrix} v \\ w \\ \varphi_x \end{pmatrix} + \\ &+ \frac{1}{2}\rho U^2 \begin{pmatrix} BKP_1^*/U & -BKP_5^*/U & -B^2KP_2^*/U \\ -BKH_5^*/U & BKH_1^*/U & B^2KH_2^*/U \\ -B^2KA_5^*/U & B^2KA_1^*/U & B^3KA_2^*/U \end{pmatrix}\begin{pmatrix} \dot{v} \\ \dot{w} \\ \dot{\varphi}_x \end{pmatrix}\end{aligned} \tag{11}$$

Where D is the drag force, L is the lift force, M is the moment; U is the wind speed, K is the reduced frequency, B is the bridge deck width, ρ is the air density, v, w, φ_x are the deck displacements and $\dot{v}$, $\dot{w}$ and $\dot{\varphi}_x$ are the velocities. A scheme of the considered aeroelastic forces acting on the bridge deck as well as the considered degrees of freedom is shown in Fig. 5.

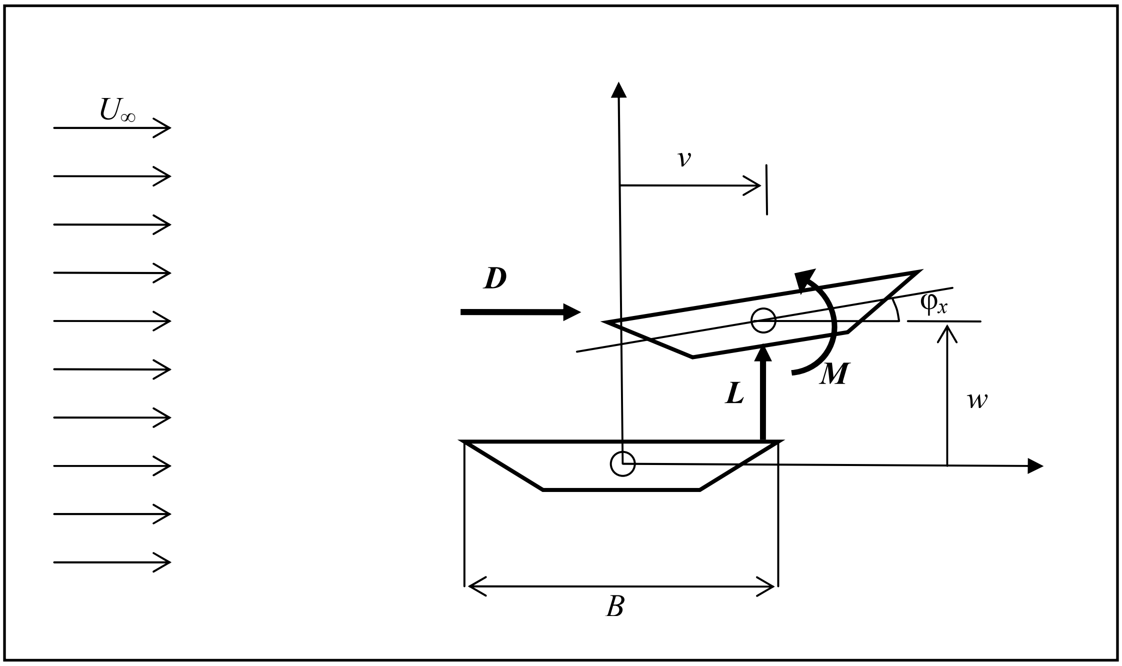

Figure 5. Displacements and aeroelastic forces on a bridge deck.

The dynamic equilibrium on the bridge deck under wind forces defined in (11) can be expressed as:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f}_{\text{a}} = \mathbf{K}_{\text{a}}\mathbf{u} + \mathbf{C}_{\text{a}}\dot{\mathbf{u}} \tag{12}$$

Where $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the stiffness matrix, $\mathbf{f}_{\text{a}}$ is the vector containing the aeroelastic forces, $\mathbf{K}_{\text{a}}$ is the aeroelastic stiffness matrix and $\mathbf{C}_{\text{a}}$ is the aeroelastic damping matrix.

The deck displacements **u** can be expressed as a combination of a set of natural mode shapes as:

$$\mathbf{u} = \mathbf{\Phi q} \tag{13}$$

Using modal decomposition and assuming a time dependent damped oscillation for the deck aeroelastic response

$$\mathbf{q}(t) = \mathbf{w}e^{\mu t} \tag{14}$$

Equation (12) leads to a nonlinear eigenvector problem

$$\left(\mathbf{A} - \mu\mathbf{I}\right)\mathbf{w}_{\mu}e^{\mu t} = \mathbf{0} \tag{15}$$

where

$$\mathbf{w}_{\mu} = \begin{pmatrix} \mu\mathbf{w} \\ \mathbf{w} \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} -\mathbf{C}_R & -\mathbf{K}_R \\ \mathbf{I} & \mathbf{0} \end{pmatrix} \tag{16}$$

In equation (16) $\mathbf{C}_R$ and $\mathbf{K}_R$ are the reduced damping and stiffness matrixes using natural mode shapes, and their definition is presented next:

$$\begin{aligned} \mathbf{K}_R &= \mathbf{\Phi}^{\mathrm{T}}(\mathbf{K} - \mathbf{K}_a)\mathbf{\Phi} \\ \mathbf{C}_R &= \mathbf{\Phi}^{\mathrm{T}}(\mathbf{C} - \mathbf{C}_a)\mathbf{\Phi} \end{aligned} \tag{17}$$

Equation (15) provides a set of pairs of conjugate complex eigenvalues, being m the number of modes considered in the analysis.

$$\mu_j = \alpha_j \pm i\beta_j \quad j=1,...,m \tag{18}$$

The nonlinear eigenvector problem must be solved for increasing values of the wind speed U until a null aeroelastic damping is obtained, that is $\alpha_j = 0$ for any j. This represents that the body motion becomes neutrally stable, being the wind speed for which the aeroelastic damping is zero the flutter wind speed U_f.

2.4 Sensitivity analysis of the flutter speed

As in the former case, several tasks must be completed in order to obtain analytically the sensitivity analysis of the flutter speed in bridges.

As modal analysis is applied in the evaluation of flutter speed, the first step in the way towards flutter sensitivity calculation is to obtain analytically the derivatives of the natural frequencies and mode shapes with regards to the considered design variables [26]. Deriving the well-known eigenvector problem it turns out:

$$\frac{\partial\left(\omega_k^2\right)}{\partial x} = \frac{\boldsymbol{\phi}_k^T \cdot \left(\dfrac{\partial \mathbf{K}_{\mathrm{nonlin}}}{\partial x} - \omega_k^2 \cdot \dfrac{\partial \mathbf{M}}{\partial x}\right) \cdot \boldsymbol{\phi}_k}{\boldsymbol{\phi}_k^T \cdot \mathbf{M} \cdot \boldsymbol{\phi}_k} \tag{19}$$

Where ω_k in the kth natural frequency $\boldsymbol{\phi}_k$ is the kth natural mode shape and **M** is the mass matrix.

Similarly, the following system must be solved to obtain the derivatives of the natural mode shapes. Special care must be taken in its solution due to its singularity.

$$\left(\mathbf{K}_{\mathrm{nonlin}} - \omega_k^2 \cdot \mathbf{M}\right) \cdot \frac{\partial \boldsymbol{\phi}_k}{\partial x} = \frac{\partial\left(\omega_k^2\right)}{\partial x} \cdot \mathbf{M} \cdot \boldsymbol{\phi}_k - \left(\frac{\partial \mathbf{K}_{\mathrm{nonlin}}}{\partial x} - \omega_k^2 \cdot \frac{\partial \mathbf{M}}{\partial x}\right) \cdot \boldsymbol{\phi}_k \tag{20}$$

The analytical evaluation of the derivative of the flutter speed with regards to a design variable x requires to derive the non linear eigenvector problem presented in Eq. (15) when α_j is null.

$$d\left[\mathbf{A}\left(\mathbf{x},U_f,K_f\right)\mathbf{w}_\mu+\frac{K_fU_f}{B}i\mathbf{I}\mathbf{w}_\mu\right]=\mathbf{0} \tag{21}$$

In Eq. (21) the dependency of matrix **A** upon a vector **x** containing the set of considered design variables as well as the flutter speed and the associated reduced frequency has been pointed out. In this research has been assumed that the external deck cross-section shape does not change. Thus, flutter derivatives are not dependent upon the set of chosen design variables.

After some mathematical manipulation and bearing in mind that the derivatives of the structure natural frequencies and mode shapes had been calculated previously, it is obtained a real number for the searched flutter speed sensitivity [17].

$$\frac{\partial U_f}{\partial x}=\frac{-\operatorname{Im}\left(\overline{g}_K h_{Ax}\right)}{\operatorname{Im}\left(\overline{g}_K g_U\right)} \tag{22}$$

The imaginary numbers h_{Ax}, h_{AU}, h_{AK}, g_U and g_K are defined as follows. The numbers $\overline{g}_U$ and $\overline{g}_K$ are the conjugate of g_U and g_K .

$$h_{Ax}=\mathbf{v}_\mu^T\frac{\partial\mathbf{A}}{\partial x}\mathbf{w}_\mu;\quad h_{AU}=\mathbf{v}_\mu^T\frac{\partial\mathbf{A}}{\partial U_f}\mathbf{w}_\mu;\quad h_{AK}=\mathbf{v}_\mu^T\frac{\partial\mathbf{A}}{\partial K_f}\mathbf{w}_\mu \tag{23}$$

$$g_U=h_{AU}+\frac{iK_f}{B}\mathbf{v}_\mu^T\mathbf{I}\mathbf{w}_\mu;\quad g_K=h_{AK}+\frac{iU_f}{B}\mathbf{v}_\mu^T\mathbf{I}\mathbf{w}_\mu \tag{24}$$

The derivatives of the **A** matrix with regards to any of the design variables can be obtained analytically as:

$$\frac{\partial\mathbf{A}}{\partial x}=\begin{pmatrix}-\dfrac{\partial\mathbf{C}_R}{\partial x} & -\dfrac{\partial\mathbf{K}_R}{\partial x}\\ \mathbf{0} & \mathbf{0}\end{pmatrix} \tag{25}$$

Evaluation of the derivative of **A** with regards to x requires previous evaluation of sensitivities of the natural frequencies and mode shapes with regards to that design variable.

The derivative with regards to the flutter speed is:

$$\frac{\partial\mathbf{A}}{\partial U_f}=\begin{pmatrix}-\dfrac{\partial\mathbf{C}_R}{\partial U_f} & -\dfrac{\partial\mathbf{K}_R}{\partial U_f}\\ \mathbf{0} & \mathbf{0}\end{pmatrix} \tag{26}$$

Operating and arranging terms it turns out:

$$\begin{aligned}\frac{\partial\mathbf{K}_R}{\partial U_f}&=-\mathbf{\Phi}^{\mathrm{T}}\frac{2\mathbf{K}_a}{U_f}\mathbf{\Phi}\\ \frac{\partial\mathbf{C}_R}{\partial U_f}&=-\mathbf{\Phi}^{\mathrm{T}}\frac{\mathbf{C}_a}{U_f}\mathbf{\Phi}\end{aligned} \tag{27}$$

Finally, the derivative with regards to the critical reduced frequency is:

$$\frac{\partial\mathbf{A}}{\partial K_f}=\begin{pmatrix}-\dfrac{\partial\mathbf{C}_R}{\partial K_f} & -\dfrac{\partial\mathbf{K}_R}{\partial K_f}\\ \mathbf{0} & \mathbf{0}\end{pmatrix}=\begin{pmatrix}\mathbf{\Phi}^{\mathrm{T}}\dfrac{\partial\mathbf{C}_a}{\partial K_f}\mathbf{\Phi} & \mathbf{\Phi}^{\mathrm{T}}\dfrac{\partial\mathbf{K}_a}{\partial K_f}\mathbf{\Phi}\\ \mathbf{0} & \mathbf{0}\end{pmatrix} \tag{28}$$

2.5 Sensitivity analysis of the deck displacements

Analytical evaluation of the gradient of the kinematic constraint considered in the optimization problem involves the calculation of the derivatives of the deck movements caused by static loads with regards to the design variables. Those derivatives are obtained by deriving the static problem in second order theory and finally, after some manipulation, the following system of equations [26] must be solved:

$$\left[\mathbf{K}_{\text{lin}} + \mathbf{K}_{\text{G}} + \mathbf{IE}\left(\mathbf{K}_{\text{G}}^{\text{b}} \cdot \mathbf{u} \cdot \mathbf{w}^{\text{T}}\right)\right] \cdot \frac{\partial \mathbf{u}}{\partial x} = \frac{\partial \mathbf{p}}{\partial x} - \frac{\partial \mathbf{K}_{\text{lin}}}{\partial x} \cdot \mathbf{u} - \mathbf{IE}\left(\mathbf{K}_{\text{G}}^{\text{b}} \cdot \mathbf{u}^{\text{b}} \cdot \frac{\partial \mathbf{w}^{\text{T}}}{\partial x} \cdot \mathbf{u}^{\text{b}}\right) \tag{29}$$

Where **IE** represents the matrix assembly operation, **p** is a vector containing the acting static loads and **w** is the vector which relates the tension force with the nodal displacements in beam elements.

2.6 Design Variables

Formulation of the optimum design problem for a long span bridge with a box deck has been presented at section 2.1. See Eqs. (4) to (7). The assumed design variables have been the thicknesses of the box cross-section. However, careful reflection is worthy to demonstrate that those design variables are the core ones governing the bridge deck aeroelastic behaviour.

The aeroelastic response of a long span bridge depends upon the geometric properties of the deck cross-section I_y, I_z, J, A as these parameters are included in the deck mass and stiffness matrixes as much in box as in truss girders. Therefore, the aforementioned geometric properties are going to play a key role in the aeroelastic performance of the bridge as can be inferred from the dynamic equilibrium equation for the bridge deck Eq. (12). Mathematically that dependency can be expressed as:

$$U_f = U_f(I_y, I_z, J, A) \tag{30}$$

Evaluation of the derivatives of the flutter speed U_f with regards to any of the cross-section geometric properties can be done by means of the formulae presented in previous points.

For box girders the geometric properties of the deck cross-section depend upon the lengths and thicknesses of the cross-section composite parts. This can be seen in figure 6, where a simplified symmetric box cross-section is shown.

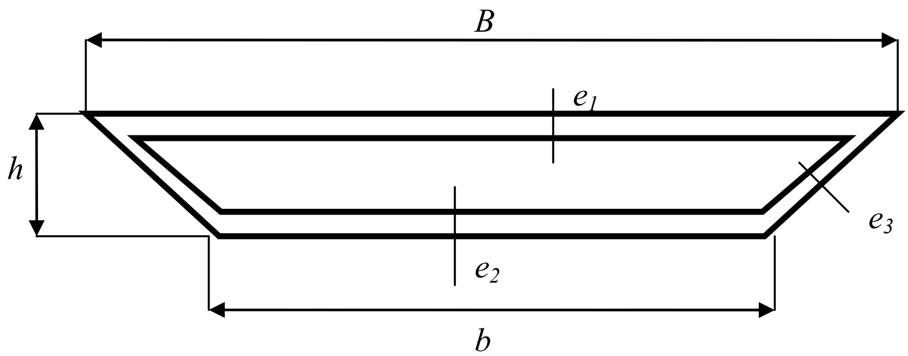

Figure 6. Simplified symmetric box cross-section.

It is assumed that the box cross-section external shape does not change along the optimum design process. That is, the lengths of the cross-section composite parts are fixed, although thicknesses are freed to change. This approach is consistent with the practice of first developing an extensive wind tunnel testing campaign in order to fulfil an efficient bridge deck aerodynamic behaviour, followed by a comprehensive design phase in which the external deck geometry is considered unchangeable. Mathematically, the dependencies of the cross-section geometric properties with regard to the thicknesses can be written as:

$$
\begin{aligned}
I_y &= I_y(e_1, e_2, ..., e_n) \\
I_z &= I_z(e_1, e_2, ..., e_n) \\
J &= J(e_1, e_2, ..., e_n) \\
A &= A(e_1, e_2, ..., e_n)
\end{aligned}
\tag{31}
$$

Thus, the flutter speed is a function of the deck cross-section thicknesses.

$$
U_f = U_f(I_y(e_1, e_2, ..., e_n), I_z(e_1, e_2, ..., e_n), J(e_1, e_2, ..., e_n), A(e_1, e_2, ..., e_n))
\tag{32}
$$

The sensitivity of the flutter speed with regards to any of the thicknesses of the cross-section composite parts is:

$$
\frac{\partial U_f}{\partial e_i} = \frac{\partial U_f}{\partial I_y}\frac{\partial I_y}{\partial e_i} + \frac{\partial U_f}{\partial I_z}\frac{\partial I_z}{\partial e_i} + \frac{\partial U_f}{\partial J}\frac{\partial J}{\partial e_i} + \frac{\partial U_f}{\partial A}\frac{\partial A}{\partial e_i}
\tag{33}
$$

A matrix expression may be used to obtain the flutter speed derivatives:

$$
\begin{Bmatrix} \dfrac{\partial U_f}{\partial e_1} \\ \vdots \\ \dfrac{\partial U_f}{\partial e_n} \end{Bmatrix} = \begin{pmatrix} \dfrac{\partial I_y}{\partial e_1} & \dfrac{\partial I_z}{\partial e_1} & \dfrac{\partial J}{\partial e_1} & \dfrac{\partial A}{\partial e_1} \\ \vdots & \vdots & \vdots & \vdots \\ \dfrac{\partial I_y}{\partial e_n} & \dfrac{\partial I_z}{\partial e_n} & \dfrac{\partial J}{\partial e_n} & \dfrac{\partial A}{\partial e_n} \end{pmatrix} \cdot \begin{Bmatrix} \dfrac{\partial U_f}{\partial I_y} \\ \dfrac{\partial U_f}{\partial I_z} \\ \dfrac{\partial U_f}{\partial J} \\ \dfrac{\partial U_f}{\partial A} \end{Bmatrix}
\tag{34}
$$

Where $\frac{\partial U_f}{\partial I_y}$, $\frac{\partial U_f}{\partial I_z}$, $\frac{\partial U_f}{\partial J}$, $\frac{\partial U_f}{\partial A}$ can be evaluated using Eq. (22). On the other hand, $\frac{\partial I_y}{\partial e_i}$, $\frac{\partial I_z}{\partial e_i}$, $\frac{\partial J}{\partial e_i}$ y $\frac{\partial A}{\partial e_i}$ with i=1,...,n must be calculated for each specific deck cross-section.

3 APPLICATION EXAMPLE

The explained formulation has been applied by the authors to the optimization problem of the Messina Strait Bridge and the results have been presented at the 4th International Symposium on Computational Wind Engineering that took place in 2006 [27]. For the structural optimization of the cross section three simplified boxes were considered (see Fig. 6), while the flutter derivatives employed in the evaluation of the aeroelastic response were the ones of the 2002 design proposal. The thicknesses of the boxes were the chosen set of design variables (e_i=e_1,…,e_6). The optimum design problem considering as behaviour constraints the critical flutter wind speed and the vertical displacements under live loads for the Messina Bridge is formulated as follows:

$$\min F = A = A^c + 2A^l = B_c h_c - \left(\frac{B_c - b_c}{2}\right) h_c - \frac{(h_c - e_1 - e_2)^2 (B_c - b_c)}{2h_c} - \\ -(h_c - e_1 - e_2)\left\{B_c - \frac{1}{h_c}\left[(h_c - e_2)(b_c - h_c) + e_3\sqrt{(B_c - b_c)^2 + 4h_c^2}\right]\right\} + \\ +2\left\{\begin{array}{l} B_l h_l - \left(\frac{B_l - b_l}{2}\right) h_l - \frac{(h_l - e_4 - e_5)^2 (B_l - b_l)}{2h_l} - \\ -(h_l - e_4 - e_5)\left\{B_l - \frac{1}{h_l}\left[(h_l - e_5)(b_l - h_l) + e_6\sqrt{(B_l - b_l)^2 + 4h_l^2}\right]\right\}\end{array}\right\} \quad (35)$$

Subject to:

$$0.003 \text{ m} \le e_i \quad (36)$$

$$g_1: \ \frac{U_{f,cr}}{U_f} - 1 \le 0 \quad \text{with } U_{f,cr} = 75 \text{ m/s} \quad (37)$$

$$g_2: \ \frac{w_c}{w_{c,\max}} - 1 \le 0 \quad \text{with } w_{c,\max} = \frac{3l}{1000} = 9.9 \text{ m; } l\text{=3300 m} \quad (38)$$

The initial design was the 2002 design proposal, thus any decrement in the deck weight compatible with the flutter and the kinematic constraints would mean a substantial saving in the project execution costs.

Several cases were solved, reaching a maximum material saving in the deck material of 33% with regards to the 2002 design proposal. Moreover, a mass redistribution between the central and the lateral boxes took place, which gives an idea of the power of this non-conventional design technique. On the other hand, improvements can be introduced in the optimization problem such as the inclusion of restraints related with stresses that would act as active constraints in the optimization problem and thus limit the magnitude of the change in the objective function.

4 CONCLUSIONS

- The feasibility of non-conventional design techniques, that is, sensitivity analysis and optimum design, has been commented; altogether with some advantages over the classical design process.
- The formulation of the optimum design problem of box girder suspension bridges considering aeroelastic and kinematic constraints has been surveyed. Moreover, the advantages of the analytical formulation for sensitivities over the finite differences approach have been pointed out.
- The formulae of the main tasks to be performed along the optimum design process have been presented.
- A reference has been included regarding the Messina Bridge application example presented by the authors in the CWE-2006 conference.
- The optimization process can be improved introducing additional aeroelastic constraints to be fulfilled such as the ones related with buffeting. Also additional structural constraints can be introduced as those related with material stresses.

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