THE EFFECT OF NONHARMONIC FORCING ON BLUFF-BODY AERODYNAMICS AT A LOW REYNOLDS NUMBER

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Abstract: The aerodynamic response of a circular cylinder to nonharmonic forcing of the inflow velocity is studied by numerically solving the two-dimensional Navier-Stokes equations on an orthogonal curvilinear mesh. The effect of varying the inflow velocity waveform while maintaining other forcing parameters constant is considered in this paper. A Reynolds number of 180 based on a reference velocity is employed in the simulations. The forcing frequency is 0.84 times the natural vortex shedding frequency in the unforced wake while the peak-to-peak amplitude of velocity oscillation is set at 65% of the reference velocity. The results show that the wake response is locked-on at the forcing frequency for all cases tested. The aerodynamic response is systematically modified by the imposed changes in the velocity waveform. The magnitude and the phase of the fluctuating drag and lift forces and the mean drag force are affected. These effects are induced by different modes of vortex formation and shedding in the wake; it is found that the rolling up of the individual shear layers on both sides can be manipulated to produce single vortices or vortex pairs.
1 INTRODUCTION

It is well known that the wake of a bluff-body behaves like a self-excited fluid oscillator above a critical Reynolds number of approximately forty five, giving rise to periodic vortex formation and shedding at the Strouhal frequency [1]. Beyond this critical point further flow instabilities lead progressively to three-dimensionality and turbulence in the wake as a function of the Reynolds number [2]. When the fluid oscillator is externally forced then the resulting wake flow is dependent on the forcing parameters as well as the Reynolds number. For harmonic forcing methods such as rectilinear cylinder oscillations at any angle to the flow direction, rotational oscillations, inflow fluctuations, etc., the response of the wake becomes a function of the forcing frequency and amplitude because the perturbation imposed can be described by a simple sinusoidal function. The study of harmonically forced bluff-body wakes has received considerable attention over the last decades with a view for flow and/or vibration control. This requires an understanding of the mechanisms of energy transfer between the fluid and the wake-generating body, which would give rise to self-excitation in the corresponding case of flexible or elastically-mounted structures. However, the use of harmonic perturbations poses a limitation on the expected range of response characteristics since the harmonic solution is an approximation to nonlinear system dynamics. For example, there is controversy whether a 2P mode of vortex shedding, i.e. where two vortex pairs are shed from alternates sides per cycle, can occur in two-dimensional conditions, or not [3]. If yes, how are the magnitude and the phase of the fluctuating forces affected by the associated changes in vortex shedding dynamics? Could such phenomena be associated with nonlinear nonharmonic dynamics? The answer to these questions forms the scope of examining the response of bluff-body wakes to nonharmonic forcing, a subject which has received scant attention so far.

The purpose of this paper is to study the effect of nonharmonic forcing on the aerodynamic and wake response of a fixed circular cylinder by means of two-dimensional numerical simulations at a low Reynolds number. In order to force the wake, the inflow velocity is given a periodic nonharmonic fluctuation,

\[ U(t) = U_0 \left[ 1 + \alpha^2 \sin^2(\omega t + \phi_0) \right]^n, \]  

\[ (1) \]

where \( U_0 \) is a reference velocity (m/s), \( \omega \) is the angular frequency (rad/s), and \( \alpha \) is a dimensionless parameter relating to the amplitude of fluctuation. The phase-shift \( \phi_0 \) is employed in order to set \( U(t = 0) = U_0 \). The index \( n \) appears as an additional parameter that determines the waveform of the imposed perturbation. For \( n = 0 \), there is no perturbation (steady flow) while for \( n = 1 \) the classical harmonic perturbation is attained (sinusoidal). Figure 1 shows the waveforms for \( n = 1, 1/2, 1/3 \) and \(-1\) employed for the present study. It should be noted that the imposed velocity waveform repeats itself twice during a full cycle \( (T_e = 2\pi/\omega) \) by virtue of Eq. (1). In order to have comparable results for different nonharmonic perturbations, the above parameters were selected so that the minimum and maximum inflow velocities were a preset factor of the reference velocity \( U_0 \). For the present study the peak-to-peak amplitude was set at \( 0.65U_0 \). The frequency of imposed fluctuation was \( 0.84f_0 \) where \( f_0 \) is the natural vortex shedding frequency in the unforced wake. Therefore, the effect of varying the index \( n \) is reported in this paper with all other parameters held constant. Essentially, the inflow velocity varies between the same minimum and maximum value but the time spend at high \((U > 1)\) and low \((U < 1)\) speeds changes with \( n \) while the period of a full cycle is the same. The Reynolds number based on the reference velocity is 180. For the analysis of the results, the aerodynamic response is determined from the computed time series of the flow-induced lift and drag forces.
exerted on the cylinder and the corresponding flow patterns are examined to determine the wake response.

2 COMPUTATIONAL METHODOLOGY

The computational methodology is a time-dependent simulation that has been previously developed and validated [4, 5, 6]. It solves the two-dimensional volume-averaged Navier–Stokes equations on an orthogonal curvilinear grid, which have the following general conservative form:

\[
\frac{\partial}{\partial t}(\rho \Phi) + \frac{1}{l_\xi l_\eta} \frac{\partial}{\partial \xi} (\rho u_\xi l_\eta \Phi) + \frac{1}{l_\xi l_\eta} \frac{\partial}{\partial \eta} (\rho u_\eta l_\xi \Phi) = \frac{1}{l_\xi l_\eta} \frac{\partial}{\partial \xi} \left( \mu \frac{l_\eta}{l_\xi} \frac{\partial \Phi}{\partial \xi} \right) + \frac{1}{l_\xi l_\eta} \frac{\partial}{\partial \eta} \left( \mu \frac{l_\xi}{l_\eta} \frac{\partial \Phi}{\partial \eta} \right) + S_\Phi
\]  (2)

where \( l_\xi \) and \( l_\eta \) are the spatially varying metric coefficients related to the orthogonal curvilinear coordinates \((\xi, \eta)\). \( \Phi = 1, u_\xi, v_\eta \) for the continuity, and the momentum equations respectively and \( S_\Phi \) are the source terms as presented in [15], including pressure terms. The original SIMPLE algorithm [7] is combined with the Rhie and Chow modification [8] for the pressure coupling on the collocated grid. A fully-implicit first-order Euler discretisation of the temporal term is used and a higher order bounded upwind scheme [9] is used for the convection terms. The rectangular computational domain extends \( 10D \) upstream, \( 25D \) downstream and \( 10D \) above and below the cylinder (\( D \) is the cylinder diameter). The inflow velocity is specified at the inlet (Eq. 1) while a convective boundary condition is employed at the outlet. Symmetry conditions are used in the lower and upper boundaries and the no-slip condition is applied at the cylinder surface. The orthogonal curvilinear mesh consists of \( 299 \times 208 \) nodes which is sufficiently dense for mesh-independent results. The time step is 0.5 ms which corresponds to 1250 steps per cycle.
3 RESULTS AND DISCUSSION

3.1 Non-forced flow

Initially, the steady unforced flow about a circular cylinder at $Re = 180$ was studied for code verification. The simulation started from an initial flow field obtained at a lower Reynolds number with periodic vortex shedding. The time histories of the aerodynamic forces exerted on the cylinder and their corresponding spectra are shown in Fig. 2. After a transient behaviour for $tU/D < 30$, a steady periodic solution is obtained. Table (1) lists the time-averaged aerodynamic properties of mean drag and fluctuating drag and lift forces after the solution has stabilised to a limit cycle, indicative of the saturation of the absolute instability in the near wake. The results agree favourably with other published data (see, e.g., the empirical fits provided in references [10, 11]).

![Figure 2: Time histories of aerodynamic force coefficients and corresponding spectra over the fully-developed periodic solution.](image)

<table>
<thead>
<tr>
<th>$Re$</th>
<th>$St$</th>
<th>$C_D$</th>
<th>$C'_L$</th>
<th>$C''_D$</th>
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</thead>
<tbody>
<tr>
<td>180</td>
<td>0.192</td>
<td>1.334</td>
<td>0.455</td>
<td>0.018</td>
</tr>
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</table>

Table 1: Computational results for unforced flow.

The spectra shown in Fig. 2 were computed over eight vortex shedding cycles for which a steady periodic solution has been attained. The drag spectra have a dominant peak at twice the natural vortex shedding frequency $2f_0$ whereas the lift spectra at $f_0$ owing to the fact that two vortices are shed from alternate sides during one full period of wake oscillation. An interesting feature of both the drag and lift spectra is that there are harmonics of the fundamental frequency. Even though these harmonics are two orders of magnitude lower than the fundamental one, their existence demonstrates the non-linearity of the system dynamics.
3.2 Effect of nonharmonic forcing

It is well known that vortex shedding lock-on occurs in bluff-body wakes when externally forced by various means whereby the shedding frequency is controlled by the forcing frequency over a range of parameters in the frequency-amplitude plane. In this study, vortex shedding lock-on is induced by forcing the inflow velocity, i.e., by superimposing an oscillatory component on a nonzero mean velocity. In the following, we investigate the case where the oscillatory component is nonharmonic and compare the results to the corresponding case of harmonic (pure-tone) forcing.

![Figure 3: Fluctuating forces acting on the cylinder over a forcing cycle; solid lines: $n = 1/2$, dashed lines: $n = 1$.](image)

3.2.1 Fluctuating forces

The effect of nonharmonic forcing on the aerodynamic forces exerted on the cylinder is shown in Figs 3 to 5 that show the instantaneous drag and lift coefficients over a forcing period. The force coefficients were computed by integration of the distributions of the viscous shear stress and pressure coefficient over the circumference of the cylinder. Solid lines indicate the results for nonharmonic forcing ($n \neq 0, 1$) in comparison to those for harmonic forcing ($n = 1$) indicated by dashed lines. All results were obtained for the same forcing frequency and same
min/max inflow velocity. For the parameters employed, the wake response is controlled by the forcing and vortex shedding occurs in all cases studied.

Figure 3 shows the force coefficients for $n = 1/2$. The inflow velocity waveform $U(t)$ for this case differs slightly from the corresponding harmonic forcing. Both the viscous and pressure components of the drag force are affected compared to the harmonic case. This should be expected because there are two force components in the direction of the flow that are dependent on the flow acceleration due to potential added mass and pressure waves (arising from velocity fluctuations) as is discussed in textbooks [12]. However, the lift coefficient depends only on the flow dynamics around the body. Therefore, the changes observed in the lift coefficient (mainly the pressure component) are due to changes in the dynamics of vortex shedding in the wake (Fig. 3). Similar but more pronounced trends to the same effect can be observed for $n = 1/3$ in Fig. 4. Deviations from the harmonic case are most pronounced for $n = -1$ where the waveform is quite different from a pure tone (see Fig. 5). As discussed in the following section, the observed changes in the aerodynamic forces are associated with corresponding changes in the dynamics of vortex shedding.
Figure 5: Fluctuating forces acting on the cylinder over a forcing cycle; solid lines: \( n = -1 \), dashed lines: \( n = 1 \).
Figure 6: Lissajous plots of the drag and lift force for different perturbation waveforms.

Figure 6 shows the Lissajous plots of the drag and lift forces coefficients after a periodic solution was attained which illustrates graphically the influence of nonharmonic forcing. For all cases, the frequency of the drag force is locked-on to twice the nominal excitation frequency while that of the lift force is locked-on to the nominal excitation frequency. This might be expected based on the case of harmonic excitation since the \( \{ \omega, \alpha \} \) combination falls within the fundamental lock-on range for sinusoidal (harmonic) cylinder oscillations or equivalent inflow fluctuations [13, 14]. This is accompanied by a magnification of the fluctuating forces exerted on the cylinder compared to the unforced flow. A notable variation in the Lissajous figures as well as in the peak/trough forces can be observed as a function of the index \( n \) (Fig. 6). For \( n > 0 \), a three-knotted figure is formed which is more pronounced with decreasing \( n \) values. For \( n = -1 \), the two knots disappear and a figure-8 pattern emerges accompanied by an increase in the drag fluctuation (9%) and a decrease in the lift (15%) whereas the mean drag increases to \( \bar{C}_D = 1.470 \), a 13%-decrease compared to the harmonically forced flow (\( n = 1 \)).

Table 2 summarizes the time-averaged properties of the aerodynamic forces on the cylinder subjected to external forcing. Positive values of the index \( n \) result in an increase of the mean drag coefficient and the r.m.s. lift coefficient compared to the harmonic case, whereas negative values of the index \( n \) have the opposite effect. The r.m.s. drag coefficient is also affected, however, these changes are mostly attributed to the changes in the acceleration/deceleration of the flow and associated in-phase force components mentioned above. It should be noted that the reference velocity \( U_0 \) was used in computing the dimensionless force coefficients and that the reference velocity does not correspond to the time-averaged inflow velocity except for the harmonic case.
3.2.2 Flow patterns

Numerical flow visualization of the wake patterns for different forcing waveforms is shown in Figs 7 to 10. Streakline patterns as observed in a frame of reference moving with instantaneous flow speed $U(t)$ are shown for six instants over a forcing cycle in order to reveal the wake topology. In this frame of reference, vortices will delineate closed paths. For harmonic forcing ($n = 1$), an asymmetric pattern with respect to the wake centreline is observed in Fig. 7. The wake consists of pairs and single vortices shed alternatively from the upper and lower side of the cylinder, respectively. This is similar to the P+S mode of vortex shedding in the terminology of [15]. It might be observed that initially a vortex pair is formed also on the lower side of the cylinder (Fig. 7b), however, the second vortex is weak and is subsequently absorbed by the initial strong vortex. On the contrary, the second vortex shed from the upper side of the cylinder is more pronounced compared to the corresponding one from the lower side and is sustained with downstream distance from the cylinder. For nonharmonic forcing with $n = 1/2$, similar wake patterns ensue as for harmonic forcing. However, the phase of vortex shedding is slightly shifted to occur earlier in the cycle. Furthermore, the secondary vortex in the pair is less pronounced compared to that for harmonic forcing. For $n = 1/3$, the secondary vortex is even less pronounced and the resulting wake pattern is close to, but not exactly, a staggered array of single vortices (2S mode). Reducing even further the value of the index $n$ for positive values might be expected to result in the classical von Kármán vortex street in the natural (unforced) wake. It is interesting to note, however, that for $n = 1$, 1/2 and 1/3, that the arrangement of the vortex street is transposed compared that of the unforced wake in the sense that the primary vortices shed from the upper side of the cylinder travel downstream below the wake centreline and vice versa. A negative value of the index ($n = -1$) has quite the opposite effect to those noted above; the second vortex separating from each side of the cylinder is stronger than the initial vortex and a pattern of vortex pairs forms in the wake (2P mode). This is probably the first numerical study that shows clearly the formation of the 2P mode in a two-dimensional domain; it highlights the important effect of nonharmonic forcing on the mechanics of formation and separation of vortices in cylinder wakes. The implications of this finding are quite important in terms of understanding vortex-induced vibration phenomena since the theoretical approach to this problem has been largely relied on the assumption of harmonic vibrations.

The patterns of vorticity distribution in the near wake for different perturbations at the instant corresponding to $U(t) \approx U_o$ (during the acceleration phase) are shown in Fig. (11). For harmonic forcing ($n = 1$), the pattern ensuing after shedding of vorticity from the cylinder has an asymmetrical structure consisting of a single positive vortex and a pair of negative vortices (this results in a mean lift coefficient different from zero). This pattern corresponds to the P+S mode which occurs at two-dimensional conditions [15]. The same pattern occurs for all $n > 0$ but the shed vortices have travelled further downstream with decreasing $n$ values. This might be attributed to the quicker vortex shedding induced by the more abrupt changes in $U(t)$ near

<table>
<thead>
<tr>
<th>(n)</th>
<th>(St)</th>
<th>(C_D)</th>
<th>(C_L)</th>
<th>(C'_D)</th>
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<td>1.475</td>
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<td>1.558</td>
</tr>
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</table>

Table 2: Computational results for forced flow.
Figure 7: Sequence of streakline patterns over a cycle for $n = 1$. 

(a) $t/T = 0$  
(b) $t/T = 1/5$  
(c) $t/T = 2/5$  
(d) $t/T = 3/5$  
(e) $t/T = 4/5$  
(f) $t/T = 5/5$
Figure 8: Sequence of streakline patterns over a cycle for $n=1/2$. 
Figure 9: Sequence of streakline patterns over a cycle for $n = 1/3$. 
Figure 10: Sequence of streakline patterns over a cycle for \( n = -1 \).
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Figure 11: Patterns of vorticity distribution in the near wake for different perturbation waveforms. Normalized vorticity contours: ±0.5, ±1, . . . ±5. Peak positive and negative contours are coded red and blue respectively.

the minimum inflow velocity, which, for harmonically forced flow, corresponds to the timing of shedding [16]. For \( n = -1 \), however, a complex pattern consisting of two pairs of vortices ensues similar to the known 2P mode [15]. Note that the follow up vortex shed from either side has a higher peak vorticity than the initial vortex shed from the same side. The flow within the vortex formation region for this case is marked by the clear formation of two vortices in the shear layer separating from the low side of the cylinder.

4 CLOSING REMARKS

A numerical study of forced flow about a circular cylinder was conducted by solving the two-dimensional Navier-Stokes equations on an orthogonal curvilinear grid using a finite-volume approach. In particular, the effect of nonharmonic perturbations in the inflow velocity on the aerodynamic and wake response is examined and compared to the corresponding harmonic case. The results indicate a gradual effect of the nonharmonic perturbations on the forces exerted on the circular cylinder (magnitude and phase) and on the vortex wake dynamics even at the low Reynolds number studied here. This is probably the first numerical study that clearly shows the formation of the 2P mode in a two-dimensional domain; it highlights the important effect of nonharmonic forcing on the mechanics of formation and separation of vortices in cylinder wakes. The implications of this finding are quite important in terms of understanding vortex-induced vibration phenomena since the theoretical approach to this problem has largely relied on the assumption of harmonic vibrations.
REFERENCES


