

# LOW-FREQUENCY VARIATIONS OF FORCE COEFFICIENTS ON SQUARE CYLINDERS WITH SHARP AND ROUNDED CORNERS

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**Abstract:** Square lighting poles with sharp or round corners have been observed to exhibit large amplitude oscillations with a frequency that is near that of their first mode. This frequency is one order of magnitude lower than that of the vortex shedding frequency for the observed wind speeds. As such, lock-in resonance between the vortex shedding and the poles motions has been ruled out as the cause of these oscillations. In this work, numerical simulations on square cylinders with different aspect ratios and cross-sectional shapes have been carried out to explore and quantify low-frequency variations of the aerodynamic forces and their sources. The results show that low frequency components are a basic aspect of the flow over finite length cylinders and are associated with the three-dimensional flow characteristics near the free end of the cylinder. This aspect is related to the loss of synchronization of the vortex shedding which, in turn, results in variations of the mean drag coefficient along the cylinder.

## 1 Introduction

Recent field observations of aluminum and steel square lighting poles have shown evidence of large amplitude aerodynamic-induced oscillations in wind speeds of about 15 to 20 m/s and which might have lead to fatigue failure of some of these poles. These oscillations have a frequency of about 1.5 Hz which is near that of the poles first mode. These poles, which can have sharp or rounded edges, are about 9.0 m high and have a side dimension of approximately 15 cm. Based on the non dimensional vortex shedding frequency  $St = f_s d / U \sim 0.13$ , it is highly unlikely that a lock-in resonance between the vortex shedding, which would have a frequency between 13 and 17 Hz for wind speeds in the range between 15 to 20 m/s, and lighting pole motions is the cause of these oscillations. This observation points to other plausible structural and/or aerodynamic coupling characteristics as responsible for the observed oscillations.

One plausible mechanism would be a structural nonlinearity that would cause energy transfer from a high frequency mode that is directly excited by the vortex shedding to the first mode.

Another plausible mechanism would be low frequency fluctuations of the aerodynamic forces which have been reported in several studies on fluid flows past bluff bodies. Roshko [1] noted that time histories of velocity fluctuations in the near wake of a circular cylinder exhibited low frequency bursts. Gerrard [2] observed the presence of fluctuations at the vortex shedding frequency and other components with frequencies that are approximately one order of magnitude lower than that of the shedding frequency. Roshko [3] suggested that low frequency fluctuations are related to the three dimensional nature of the flow, and may be caused by loss of spanwise coherence of the shed vortices. This loss may be due to intrinsic or extrinsic effects. The former effects are related to natural flow instabilities while the latter effects are due to geometrical features, which make the flow three dimensional (end effects, cylinder’s motions, etc.). Lisosky [4] observed the presence of the low frequency fluctuations in the drag and lift coefficients as measured on circular cylinders, and noted that the amplitude of these fluctuations are affected by the cylinder’s length to diameter aspect ratio. Miao et al. [5], noted the presence of low frequency fluctuations in the pressure signals measured in the wake of trapezoidal cylinders, suggesting that low frequency fluctuations are also present in flows past cylinders having non-circular section shapes.

The overall goal of our effort is to develop a comprehensive understanding of the physical mechanisms behind observed large amplitude oscillations of square lighting poles. In this study, three-dimensional numerical simulations of the aerodynamic flow past square cylinders under different conditions are conducted. To isolate the role of end effects in the generation of low frequency components in the aerodynamic forces, we compare characteristics of force coefficients from simulations on cylinders that have different length,  $L$ , to side dimension,  $B$ , ratios. Particularly, force coefficients from simulations on infinite length square cylinders ( $L/B = \infty$ ) are compared with coefficients from simulations on finite length cylinders ( $L/B = 60$ ). These simulations are carried out for both sharp and rounded edges,  $r/B = 0$  and  $1/12$ , respectively, where  $r$  is the corner radius. Time series and spectral analysis of the drag and lift force coefficients are then used to determine the fluctuation characteristics in all cases. Additionally, there has been some inconsistencies in reported values for the mean drag coefficient on square cylinders with sharp corners. While most of the reported values are near 2.0, there have been other reported values near 1.5 ([6], [7]). As such, we will also discuss the effects of different simulation conditions on the mean drag values for square cylinders with sharp and rounded corners.

## 2 Numerical simulations

A series of numerical simulations of the unsteady flow past square cylinders with sharp and rounded corners at  $Re = 100000$  was conducted to isolate the role of end effects, if any, in the generation of low-frequency components in the force coefficients. To this end, the unsteady incompressible Reynolds Averaged Navier–Stokes (RANS) equations, complemented by a  $k-\omega$  two equations closure model are solved. These equations are solved by means of the commercial software Fluent employing a finite volume discretization method. The computational domain is a 10 m x 3 m x 1.8 m. The cylinder is a long prism with a square cross section that has a side dimension  $B = 0.02$  m. To isolate the end effects, we consider two different cylinder setups in the numerical simulations. In the first configuration, the cylinder is 1.8 m long with both ends attached to opposing surfaces of the computational domain; thereby, eliminating any end effects. In a second configuration, the cylinder is 1.2 m long and is mounted on one side of the computational domain with the other end free. In this case, the length of the cylinder is set at  $L = 60B$ . In the following, we will refer to these two configurations as *infinite length* and *finite*

*length* cylinders, respectively.

A no slip boundary condition is applied on the cylinder surface. The sides of the computational domains were considered slip walls. Turbulence intensity and viscosity ratio values of 1% and 10 respectively were imposed at the inflow boundary. At the outflow boundary, a null normal component of the stresses tensor was imposed. A hybrid mesh composed of prismatic elements in the cylinder boundary layer and of tetrahedral elements in the rest of the domain was generated for the computations in order to adequately simulate the high gradients in the near wall region. Localized refinements in the wake region have been also implemented. The number of degrees of freedom for the *infinite length* cylinder case was about 700000. This number was increased to about 1500000 in the *finite length* cylinder simulations to refine the mesh in proximity of the tip.

### 3 Results and discussion

#### 3.1 Force coefficients on square cylinder with sharp corners

A comparison of time histories of the drag and lift coefficients,  $C_D$  and  $C_L$ , on the infinite and finite length cylinders with sharp corners is presented in Fig.1. The plots show several differences that can be associated with end effects. First, the mean drag coefficient for the infinite length cylinder is about 2.2. In comparison, the mean drag value on the finite length cylinder is about 1.61. Additionally, the RMS values for both lift and drag coefficients are consistently lower in the finite length case. The RMS value of the drag coefficient on the finite cylinder is 0.13 which is about 20% lower than the corresponding value, 0.16, on the infinite length cylinder. The RMS value of the lift coefficient on the finite length cylinder is 0.12 which is significantly lower than the corresponding value, 1.62, as computed on the infinite length cylinder. Finally, the time histories of the force coefficients in the finite cylinder case exhibit low-frequency variations that are not present in the infinite length case.

Power spectra of the time series of Fig.1 are presented in Fig. 2. The spectra of the lift coefficients for both infinite and finite cylinder cases exhibit a strong peak at the vortex shedding frequency given by the nondimensional Strouhal number  $St = 0.137$ . This value is in good agreement with values reported by Tamura et al. [8] and Tamura and Miyagi [9]. Smaller peaks

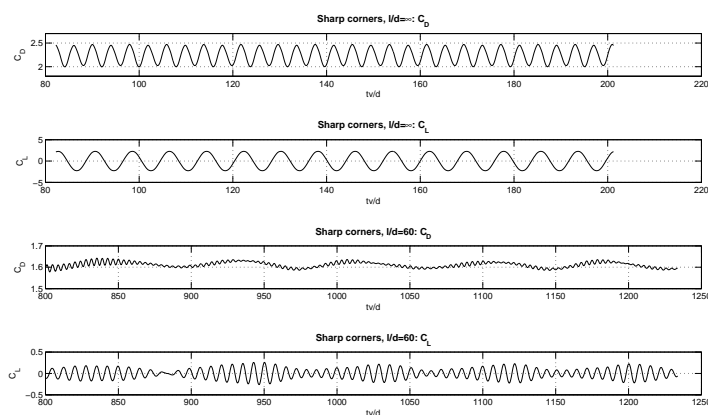


Figure 1: Time series of lift and drag coefficients on infinite length (top plots) and finite length (bottom plots) cylinders with sharp corners.  $Re = 100000$ .

at the odd harmonics of the vortex shedding frequency are also noted in the spectra of the lift coefficient. The spectra of the drag coefficients exhibit a strong peak at the second harmonic of the vortex shedding frequency. They also exhibit peaks with much smaller amplitudes at the even harmonics. The presence of peaks at the vortex shedding frequency and its harmonics in the lift and/or drag spectra is associated with the formation of an alternating vortex shedding pattern behind the cylinder. Yet, the most noticeable difference between the spectra of the force coefficients on the finite and infinite length cylinders is the presence of a low frequency peak in the drag coefficient in the case of the finite cylinder. This peak is at a frequency which is about one order of magnitude smaller than that of the vortex shedding frequency.

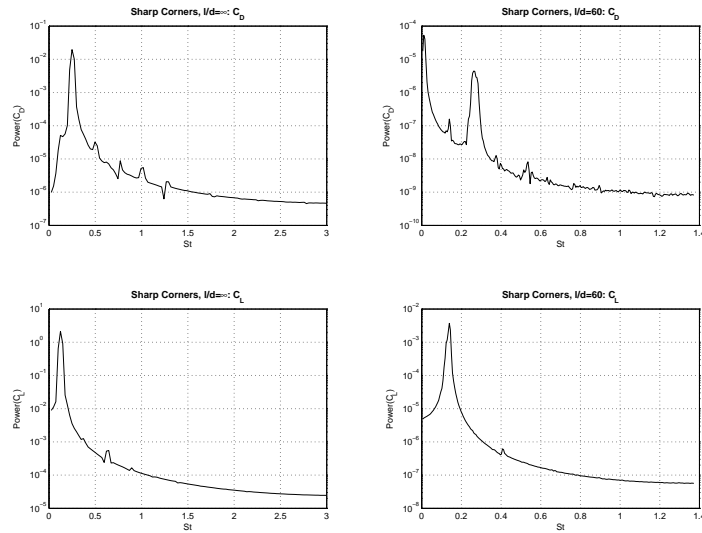


Figure 2: Power spectra of lift and drag coefficients on infinite length (left plots) and finite length (right plots) cylinders with sharp corners.  $Re = 100000$ .

The lower force coefficients and the presence of a low-frequency variation in the case of the finite cylinder are related to a loss of synchrony in the vortex shedding mechanism. This is best illustrated in Fig. 3, which depicts the pathlines of the flow field past the pole. Clearly, the flow is mostly two dimensional in proximity of the pole root (in the lower part of the figure). On the other hand, it becomes completely three dimensional near the cylinder tip. Consequently, the vortex shedding at different positions along the cylinder is not in phase and undergoes low-frequency variations which can be associated with the three dimensional characteristics of the flow near the tip. The effects of the synchronization loss of the vortex shedding along the finite length cylinder on the force coefficients can be determined from assessing the variations of these coefficients over different sections of the pole. The variation of the mean drag coefficient along the cylinder is presented in Fig.4. The results show that the mean value of the drag coefficient near the root of the cylinder is about 1.9 and decreases to a value near 1.3 at the tip of the cylinder. These observed variations in the mean and in the frequency of the force coefficients along the finite length cylinder should be carefully considered in the design for both static and dynamic wind loading on lighting poles.

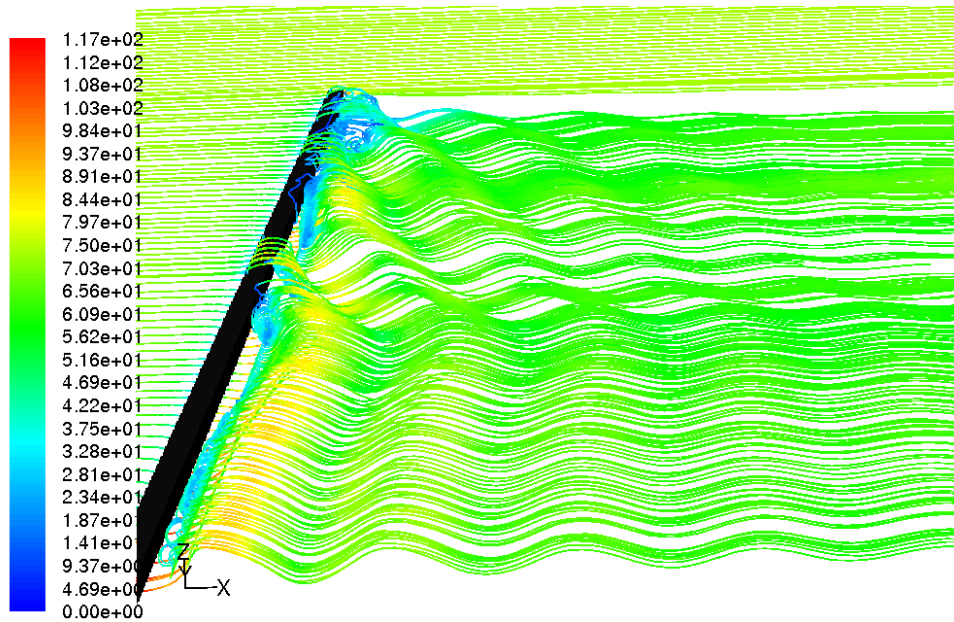


Figure 3: Pathlines of the flow past the square cylinder with sharp corners and having an aspect ratio of 60:1.  $Re = 100000$ .

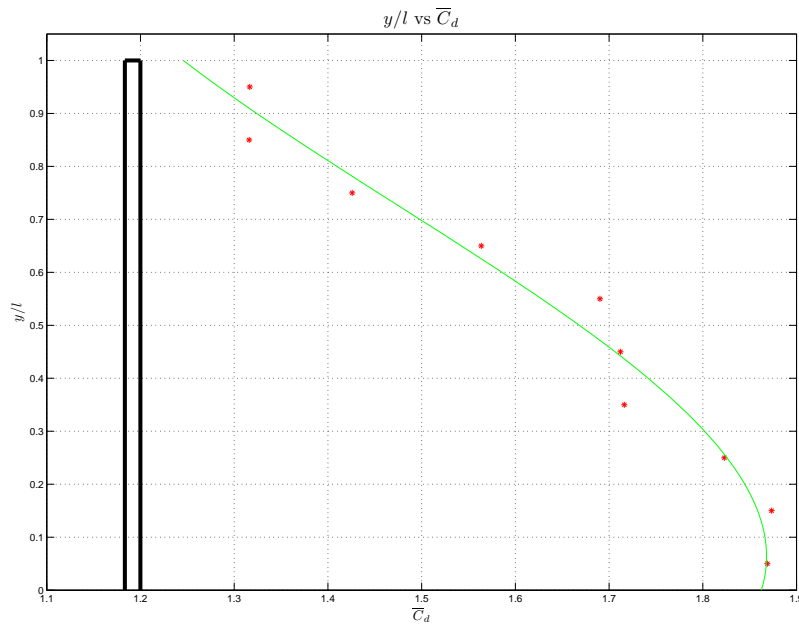


Figure 4: Variation of the mean drag coefficient along the square cylinder with sharp corners and having an aspect ratio of 60:1.  $Re = 100000$ .

### 3.2 Force coefficients on square cylinder with rounded corners

In this section, we consider the effects of rounding the cylinder corners on the mean and RMS values of the force coefficients and on their frequency content for both infinite and finite cylinder cases and on the variations of these properties along the cylinder in the finite length case. Fig. 5 shows a comparison between the time histories of the force coefficients obtained

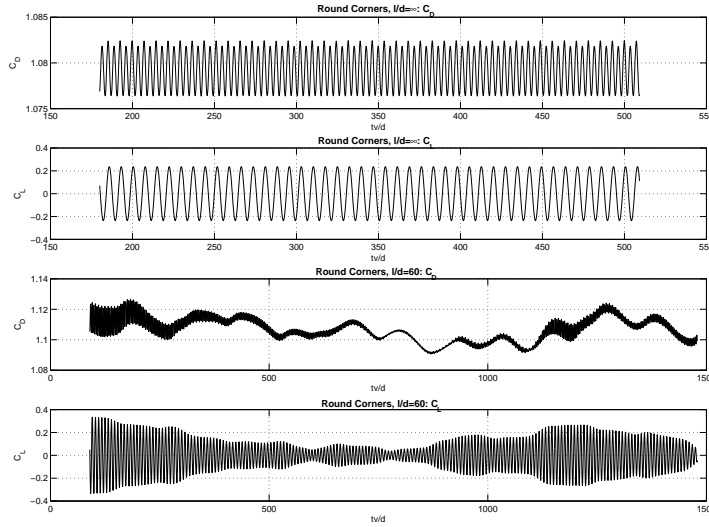


Figure 5: Time series of lift and drag coefficients on infinite length (top plots) and finite length (bottom plots) cylinders with rounded corners.  $Re = 100000$ .

for infinite and finite length square cylinders with rounded corners. The plots show that the mean drag coefficient is much smaller than their counterparts in the sharp corner cases. For the infinite length, the mean  $C_D$  value is 1.1 which is about 50% smaller than the equivalent value of sharp edges cylinders. The finite length cylinder with rounded corners has a drag coefficient of about 1.1, which is approximately 60% of the equivalent value for the square cylinder with sharp corners. It is interesting to note here that the mean drag coefficients for finite and infinite cylinders with rounded corners are about the same. This stands in contrast with the difference observed in the sharp corners case where a significant drop in the mean drag coefficient was associated with end effects in the case of the finite length cylinder. These values and differences are in agreement with trends and measurements reported by Tamura and Miyagi [9] (see their figure 8).

As in the case of sharp corners cylinders, the low frequency variations in the time series of force coefficients on finite length cylinders with rounded corners is the most appreciable difference related to end effects. Power spectra of the lift coefficients on the finite and infinite length square cylinders with rounded corners, presented in Fig. 6, are characterized by a peak at  $St = 0.137$  which corresponds to the vortex shedding frequency, along with lower peaks at the odd harmonics. As expected, the drag coefficients spectra, also presented in Fig. 6, have strong peaks at the second harmonic of the vortex shedding frequency, and a series of lower peaks at the even harmonics. Additionally, the spectrum of the drag coefficient for the finite length cylinder shows high amplitudes over low-frequency components over a range up to about  $1/10$  of the vortex shedding frequency.

The presence of low frequency fluctuations of the aerodynamic forces can again be associated with the presence of three dimensional effects which weakens the synchrony of the vortex shedding. This is illustrated in Fig. 7, which depicts the pathlines of an instantaneous flowfield past the finite length cylinder with rounded corners. Clearly, the flow is two-dimensional near the root of the cylinder and becomes three dimensional near its tip. Consequently, the obvious loss of spanwise coherence of the shedding vortices and the associated low-frequency variations

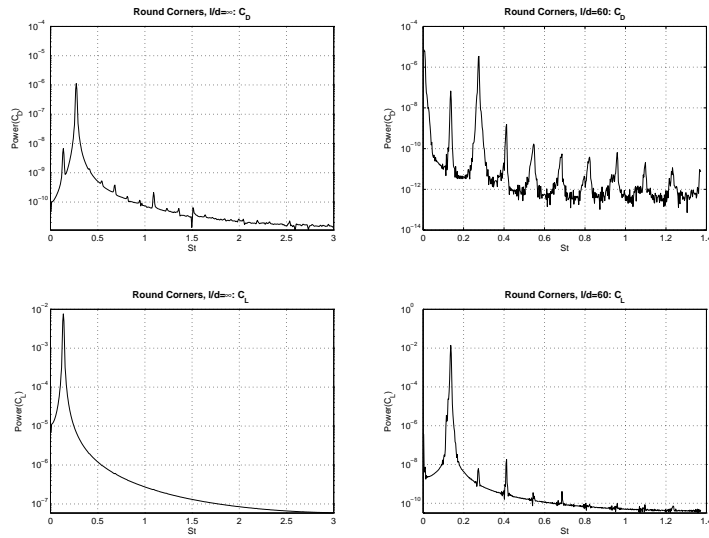


Figure 6: Power spectra of lift and drag coefficients on infinite length (left plots) and finite length (right plots) cylinders with rounded corners.  $Re = 100000$ .

are due to the tip effects. The effects of this loss of the vortex shedding synchronization on the mean drag coefficient are illustrated in Fig. 8 which shows the variation of the mean drag coefficient along the square cylinder with rounded corners. The mean drag coefficient drops from a value of about 1.15 at the cylinder root, to approximately 0.9 at the tip. This drop is not as significant as it is in the sharp corners case.

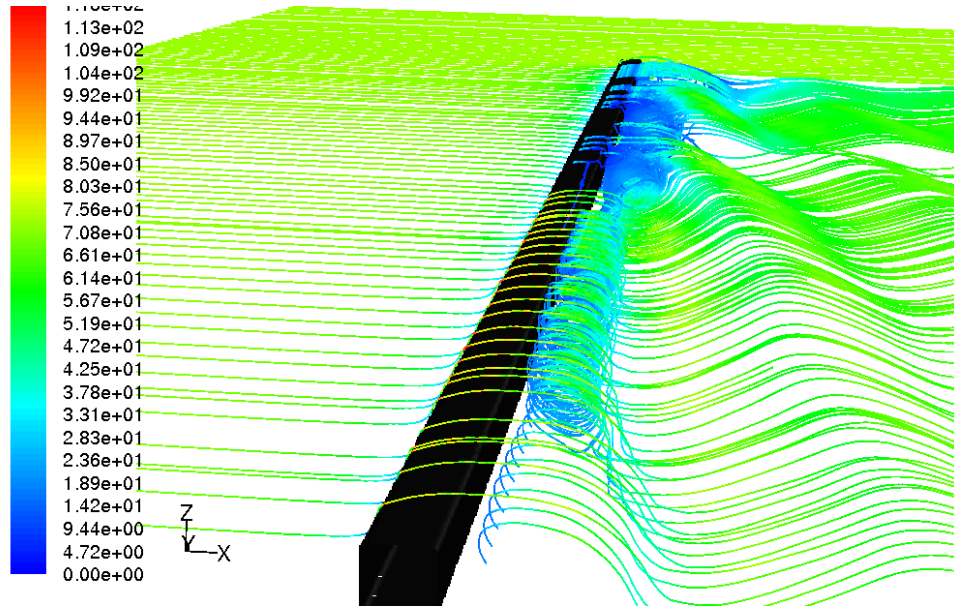


Figure 7: Pathlines of the flow past the square cylinder with rounded corners and having an aspect ratio of 60:1.  $Re = 100000$ .

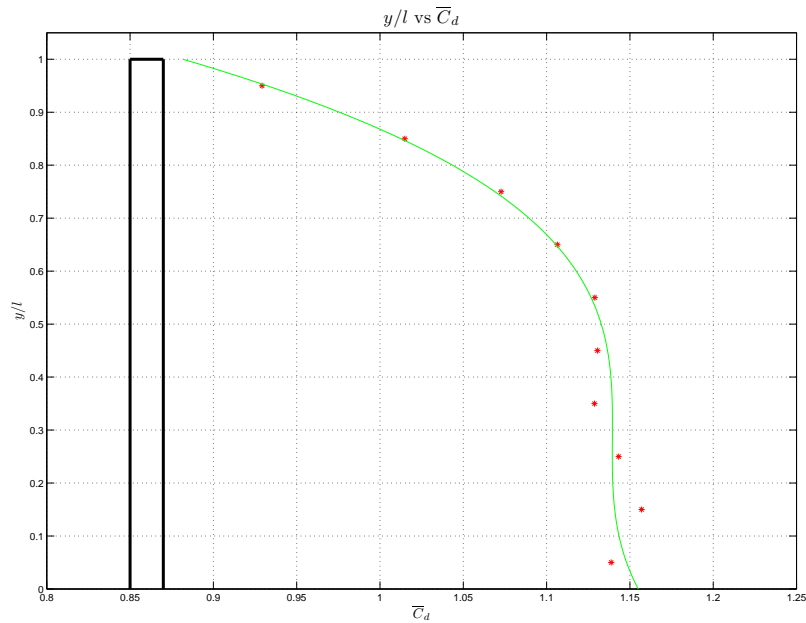


Figure 8: Variation of the mean drag coefficient along the square cylinder with rounded corners and having an aspect ratio of 60:1.  $Re = 100000$ .

#### 4 Conclusions

In this work, numerical simulations of the flow field past square cylinders with sharp or rounded edges and with different aspect ratios have been carried out to explore the presence of low frequency fluctuations in the force coefficients. The results show that low frequency components are a basic aspect of the flow over finite length cylinders and are associated with the three-dimensional flow characteristics near the free end of the cylinder. These, in turn, cause a loss of synchronization of the vortex shedding behind the cylinder. This loss of synchronization results in variations of the mean drag coefficient along the cylinder. These variations are more significant in the case of cylinders with sharp corners than in the case of those with rounded corners. Such variations in the mean and in the frequency of the force coefficients along the finite length cylinder should be carefully considered in the design for both static and dynamic wind loading on lighting poles.

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