WAKE ENERGY REDISTRIBUTION DUE TO TRAILING EDGE SPANWISE PERTURBATION

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**Abstract.** The three dimensional near wake flow topology behind a flat plate subjected to periodic spanwise sinusoidal perturbation (SSP) at the trailing edge is investigated and compared to a baseline case with no SSP using particle image velocimetry (PIV) and high speed pressure transducers installed in the base at a Reynolds number of 2.5x10\(^4\). The underlying large scale structure of the near wake is extracted using the Proper Orthogonal Decomposition (POD) technique. When compared to the baseline wake structure, SSP significantly modifies the near wake structure and strongly controls the three dimensional vortices. It also redistributes the relative energy among different modes, enhancing the streamwise vortices and suppressing the von Karman-Bernard rolls.
1 INTRODUCTION

Research into control of vortex shedding in the wake of the bluff bodies by both active and passive means has been investigated extensively. Passive control is reliable, because of simplicity and cost effectiveness, and hence is often preferred over active control methods. Successful control of vortex shedding in bluff bodies has been obtained with splitter plates, three dimensional disturbances, and surface protrusions. Periodic geometrical distortion, when applied to the trailing edge and/or the leading edge of a wake generator, suppresses the vortex shedding, thereby reducing the oscillating lift and drag on bluff bodies.

Wake flows behind nominally two dimensional bodies are three dimensionally unstable and are governed by two distinct modes of instabilities, depending on the flow Reynolds number based on base height. These two instability modes are characterized by two main coherent structures, spanwise Karman vortices and streamwise vortices referred to as rolls and ribs respectively. It has been observed that the ribs wrap around the rolls and they are interconnected. Enhancing three dimensional instabilities could lead to early suppression of the spanwise Karman vortices accompanied by fluctuating lift and base drag reduction.

Figure 1 shows these two distinct instability modes, commonly referred as Mode A and Mode B, have been extensively investigated for cylinder flows over a wide range of Reynolds numbers. The X, Y and Z represent the streamwise, transverse and spanwise coordinates. Williamson showed that Mode A occurred at $\text{Re}_D > 180$ and is gradually replaced by Mode B at $\text{Re}_D > 230$, and both modes coexist at $\text{Re}_D = 220$. The spanwise wavelength ($\lambda_z$, defined as the spacing between the same sign vortices) between streamwise vortices of Mode A is scattered between 3D to 5D, but for Mode B it has been demonstrated consistently to be around 1D in various experiments for various $\text{Re}_D$. An interesting observation was made by Brede et al., who experimentally showed that, at low $\text{Re}_D$ (160-500), Mode B peaked in circulation strength two diameters downstream of the wake, whereas Mode A peaked in circulation strength around six diameters downstream, indicating that Mode B structure dominates the near wake dynamics.

Despite the practical importance in industrial flows, square cylinder and flat plate near wake flow topology did not draw as much attention as circular cylinders. Robichaux et al. conducted low $\text{Re}_D$ numerical simulation based on Floquet analysis and found same natural modes with larger spanwise spacing of 5.22D for Mode A and 1.2D for Mode B. Vickery, based on high $\text{Re}_D$ (4 x $10^6$ to 1.6 x $10^7$) base pressure measurements proposed spanwise correlation length of 5.6D (smooth stream) and 3.3D (turbulent stream). Dobre and Hangan investigated the intermediate wake flow topology and proposed a structure similar to Mode A with a spanwise spacing of 2.4D at $\text{Re}_D = 2.2 \times 10^4$.

Julien et al. studied the dynamics of three dimensional secondary instabilities developing in the wake of a thin flat plate experimentally at a $\text{Re}$ of 200. They found Mode B structure dominated the near wake and occurred with a spanwise wavelength $\lambda_z/\lambda_x=1$, whereas in the far wake both modes coexist and grow equally. Further they investigated the response of wake...
to spanwise periodic perturbations at trailing edge and found that $\lambda_z/\lambda_x=0.76$ perturbation grew much faster than all others. El-Gammal and Hangan\textsuperscript{19} found a structure similar to Mode B for mean wake of divergent trailing edge and showed the robustness of this topology for both inflow conditions and stall incidence angles.

The present investigation seeks to better understand the near wake evolution and the effect of spanwise sinusoidal perturbations (SSP) on blunt trailing edge profiled bodies. Specifically, the changes in the wake structure and dynamics that occur when SSP is applied are investigated here. To carry this out, velocity measurements are made using PIV in the base region of a profiled flat plate with two different trailing edges. The Proper Orthogonal Decomposition (POD) is then used to identify changes in the wake structure associated with coherent energy re-distribution due to passive control.

2 EXPERIMENTAL SETUP

2.1 Facility and Model

All the measurements for this investigation are performed in the 0.61 m x 0.61 m x 1.21 m subsonic wind tunnel facility at the University of Wyoming Aeronautics Laboratories. This is an open loop wind tunnel with a variable frequency driven motor capable of producing free-stream velocities between 10 and 50 m/s. The inlet section of the wind tunnel has a honeycomb insert and three sets of screens to break down large scale non-uniformities.

For this investigation two flat plate models with a modified elliptical leading edge for smooth flow transition over the model are used. On one of these models, a spanwise sinusoidal perturbation with $\lambda_z/D = 2.4$ is applied and is referred to as the control case. In contrast, the other model has no spanwise sinusoidal perturbation and will be referred to as the base case. A 6.25 mm wide sand strip is applied near the leading edges of these models to ensure that the boundary layer is fully turbulent. For this investigation, the $Re_D$ is $2.5 \times 10^4$ or $Re_L$ (Reynolds number based on the length) is $3.1 \times 10^5$. The main characteristics of the flat plate model and the coordinate system are shown in figure 2.

![Figure 2. Flat plate model used for this investigation, along with schematic of the region in the near wake of the model imaged by the PIV system](image-url)
2.2 Instrumentation

Particle Image Velocimetry (PIV) measurements are performed in the near wake for both base case and the SSP control case to study the near wake secondary instabilities. To study the spanwise vortices PIV measurements are performed at two different vertical planes for the base case model and at three different vertical locations \((Z = 0\lambda_z, 0.25\lambda_z, \text{ and } 0.50\lambda_z)\), where \(\lambda_z\) is the wavelength of sinusoidal perturbation) for the control case. In order to capture the signatures of streamwise vortices, measurements are also performed at two different horizontal planes \((Y = 0, \text{ and } Y = D/2)\). A LaVision PIV system with a dual frame CCD camera is used for this investigation. To perform PIV measurements, the flow is seeded using atomized oil, and the wake is illuminated using twin 50 mJ Nd: YAG lasers. The wake velocity field is determined from raw images using a cross-correlation, multi-pass, decreasing window size technique. A schematic of the near wake location where PIV measurements are performed relative to the flat plate model is shown in figure 2.

To measure pressure fluctuations in the base region, three Entran (EPE-541-2P) and Endevco (8507C) high speed pressure transducers were used. Validyne model SG297A modules provided excitation and signal conditioning for these pressure transducers. Using a high speed simultaneous data acquisition system (Kinetic Systems V200), voltage signals from these transducers were acquired at a sampling frequency of 6250 Hz and filtered at 3000 Hz. The transducers location is shown in figure 2. For the reminder of the paper, the pressure transducers will be referred to by the numbering shown in figure 2. All these high-speed transducers were calibrated using a Validyne DP-15 transducer every day before the start of testing.

In order to perform the POD analysis, randomly acquired wake velocity flow field data was required. To achieve this, a soft trigger was generated randomly and was sent to both the data acquisition system and the PIV timing unit. For all experiments, 1000 random PIV images were acquired.

3 PROPER ORTHOGONAL DECOMPOSITION

POD is a mathematical approach based on the Karhunen-Loeve expansion. Let \(\vec{U}^k\) be a typical velocity vector field, and is a function of \(x\) and \(t\) \((\vec{U}^k = \vec{U}(x,t))\). POD yields a set of orthogonal modes or Eigenvectors \(\Phi\). \(\Phi\) is chosen to maximize the average projection of \(\vec{U}^k\) onto \(\Phi\), suitably normalized to optimize the kinetic energy,

\[
\frac{\left\langle \left| \left\langle \vec{U}^k, \Phi \right\rangle \right|^2 \right\rangle}{\| \Phi \|^2} \tag{1}
\]

where \(| . |\) denotes the modulus, \(\langle . \rangle\) is the ensemble average, \(( . . )\) represents the inner product and \(|| . ||\) denotes the \(L^2\) norm. This results into an Euler Lagrangian integral equation,

\[
\int_{\Omega} \langle \vec{U}(x,t_i) \otimes \vec{U}(x',t_i) \rangle \Phi(x') dx' = \lambda \Phi(x) \tag{2}
\]

where \(\Omega\) is the domain of interest, and \(\langle \vec{U}(x,t_i) \otimes \vec{U}(x',t_i) \rangle\) is an ensemble average of the dyadic product of velocity vector. \(t_i\) is PIV-snapshot time or \(k\)th PIV image, and \(\lambda\) are Eigenvalues, representing the kinetic energy associated with each of the POD modes. After discretization and numerical integration, equation can be recasted as an eigenvalue problem

\[
A \Phi = \lambda \Phi \tag{3}
\]
where $A$ is the kernel of POD formulation.

Once the POD modes are determined, the fluctuating coefficients for a specific instant in time are determined by projecting the fluctuating velocity on the POD modes.

$$a_i(t_k) = (\hat{U}(x,t_k), \Phi^i(x))$$

where $x$ denotes the projection is performed on the entire space at once. For the present analysis, the computationally more efficient method formulated by Sirovich,\textsuperscript{20} method of snapshots is implemented for $u$ and $v$ components combined in vector format. Finally the Hilbert-Schmidt theory assures that if a random field occurs over a finite domain, an infinite number of orthonormal solutions can be obtained to express the original random field $\hat{U}(x,t_k)$. Using these POD modes the velocity field can be partially or totally reconstructed\textsuperscript{21} using

$$\hat{U}(x,t_k) = \sum_{i=1}^{N_u} a_i(t_k) \Phi^i(x)$$

where $a_i(t)$ is the time varying coefficient for the $i^{th}$ POD mode $\Phi^i$ at time $t_k$, $N_u$ is the number of modes used to reconstruct the velocity field. If $N_u$ is chosen to be $N_s$ (total number of snapshots), the measured velocity field is completely reconstructed. This results in an optimal low dimensional representation of the unsteady kinetic energy contained in the flow.

4 BASE CASE RESULTS

4.1 Vertical Plane (XY)

Figure 3 show the first five modes of streamwise and transverse velocity components in the vertical plane (XY). This plane captures the signature of the spanwise von Karman-Bernard vortices or rolls. The modes indicate that they are acting in pairs, first reported by Deane et al.\textsuperscript{22} and subsequently by Noack et al.\textsuperscript{23} for low Reynolds number cylinder flow and by Durgesh\textsuperscript{24} for flat plate model at high Reynolds number. The first two modes can be combined to produce the Karman vortices at any phase in the shedding process. Similarly modes 4 and 5 contribute to the higher harmonic of Karman vortices. Mode 3 appears to be a standing mode with no companion mode. The streamwise spacing ($\lambda_x/D$), which is defined as the spacing between the vortices of same sign, is found to be 4.18 (calculated using shedding frequency of

![Figure 3. Streamwise and transverse velocity components for the first five POD modes in vertical plane for the base case](image-url)
160 Hz obtained from the base pressure spectrum and assuming convection velocity of 0.85U₀ based on Taylor’s hypothesis that all structures are advected at a constant phase).

4.2 Horizontal Plane (XZ)

Figure 4 shows the first eight modes of streamwise and spanwise velocity components in the horizontal plane (XZ). This plane captures the signature of both rolls and ribs. The first two dominant modes of the streamwise velocity component (figure 4(a)), combined indicate the Karman vortices. The next two dominant modes (3 and 4) seem to indicate the vortex dislocations in the Karman vortices caused by the streamwise vortices wrapping around the Karman vortices. Modes 5 and 6 display a signature very similar to Mode A, with alternating structures of opposite sign in the spanwise direction and with same sign in the streamwise direction (refer to figure 1 for general description of Mode A and Mode B), whereas modes 7 and 8 seems to indicate Mode B signature with structures alternating with opposite signs in both streamwise and spanwise directions. More clarity to the above argument can be added by combining the modes. Figure 5 show the streakline plots of the combined modes with streamwise and spanwise velocity contours in the background. Clearly Modes 5 and 6 seems to show a structure similar to Mode A and Modes 7 and 8, that of the structure Mode B.

Hence it appears that Mode A and Mode B appear to coexist in the near wake structure at high ReD, opposed to Mode B structure dominating the near wake as proposed by Julien et
al.\textsuperscript{18} for low \(\text{Re}_D\). The spanwise spacing (\(\lambda_z/D\)) of the streamwise vortices is found to be 4.8 for both Mode A and Mode B and the ratio between spanwise to streamwise spacing (\(\lambda_z/\lambda_x\)) is found to be 1.14. The value is inline with the value proposed by Julien et al.\textsuperscript{19} of \(\lambda_z/\lambda_x = 1\) for Mode B.

5 CONTROL CASE RESULTS

The spanwise sinusoidal perturbation used in the present experiment is originally developed by Dobre et al.\textsuperscript{8,17} based on the observation of intermediate wake flow topology of square cylinder. They successfully demonstrated that vortex shedding could be completely destroyed by using the control (SSP) at the stagnation face of the square cylinder. The study was further extended to airfoil bodies by El Gammal and Hangan.\textsuperscript{25}, and they observed suppression of vortex shedding. The same SSP control is used here in the current experiments.

![Figure 6. Streamwise and transverse velocity components for the first five POD modes in vertical plane at (a) Maximum (b) Median and (c) Minimum locations for SSP control case](image.png)
5.1 Vertical Plane (XY)

Figure 6 show the streamwise and transverse velocity component mode shapes at different vertical locations of the SSP \((Z/D = 0.05, 0.25, 0.5\lambda_z)\). At a first look, the first two modes clearly indicate the Karman vortices at all the three vertical locations. There is no significant change in the transverse velocity component mode shapes compared to the base case, however the modes shapes of the streamwise component have significantly changed in terms of separation in the transverse direction at median (Figure 6(b)) and minimum (Figure 6(c)) locations, indicating that the structure generated from these modes will be wider than the base case. Higher modes indicate that the structure tends to be more organized at the minimum location and transforms to a disorganized state as we proceed towards the maximum location of the SSP control.

![Figure 6](image)

Figure 6. (a) Streamwise and (b) Transverse velocity components of the first eight modes in vertical plane-1 for the control case

5.2 Horizontal Plane (XZ)

Figure 7 shows the first eight modes of streamwise and spanwise velocity components in the horizontal plane (XZ). The first two dominant modes of the streamwise component indicate the vortex dislocation caused due to the spanwise periodic forcing of the flow by SSP control. The next two dominant modes 3 and 4 seem to indicate the Karman vortices. Mode 5 appears to show a signature similar to Mode A. Mode 6 and 7 seem to be dislocations in Karman vortices caused by either Modes A or B or combined. The first two modes of the spanwise velocity component indicate Mode B structure caused due to the forced perturbation of the flow by SSP control. The other modes appear to be inline with what is discussed above. The spanwise spacing \((\lambda_z/D)\) of the streamwise vortices is found to be 3.73 and the ratio between spanwise to streamwise spacing \((\lambda_z/\lambda_x)\) is found to be 1.04. The values proposed by
Julien et al.\textsuperscript{18} for thin flat plate suggest that at $\lambda_z/\lambda_x = 1$, the growth rate of the disturbance is minimal.

6 COMPARISON BETWEEN BASE CASE AND CONTROL CASE

Figure 9 shows the relative energy captured by the first 10 POD modes in all vertical planes for the base case compared with the control case. POD modes are same at all vertical in the base case. Note that the flow structure is dominated by first two modes in the XY plane, i.e. spanwise vortices. It shows that Karman vortices carry about 60% of the total energy for the base case. However in the horizontal plane (Figure 10), which better captures the influence of the streamwise vorticity, several modes are involved suggesting a more even energy distribution between the rolls and rib structures.

When the control is applied, in the horizontal plane the first two modes show an increase in relative energy compared to the base case suggesting a predominant structure due to the perturbation applied. At the same time in the vertical plane, which captures the spanwise vorticity, shows a decrease of relative energy in the first two modes compared to the base case. This behavior tends to indicate that, the SSP control strengthens the streamwise vorticity and weakens the spanwise vorticity.

The above argument can be augmented by looking at the surface pressure spectrum at various locations (see Figure 2 for pressure transducer location) on the trailing edge of the flat plate. Figure 11 clearly indicates that the intensity of the von Karman vortices is reduced when SSP is applied. Also note that, when SSP is used higher harmonics of the Karman vortex shedding frequency are completely eliminated, indicating that the strength of Karman vortices is reduced. However there is not much change in the base drag with control, as it reduced only about 2% drag compared with the base case.

7 CONCLUSION AND FUTURE WORK

The modes shapes tend to indicate that in the near wake both Mode A and Mode B structure coexist. The spacing ratio between the spanwise arrangement of ribs and the streamwise rolls is found to be $\lambda_z/\lambda_x = 1.14$ for the unforced case. The SSP control applied herein indi-
cates that the strength of the Karman vortices is reduced. However, because the wake secondary instability modes are not fully triggered, results in minimal effect in terms of drag reduction. The next step in developing an effective control strategy is to perturb the flow with spanwise spacing ratio corresponding to the unforced flow case and eventually to find how sensitive this spacing ratio is dependent on the wake generator, Strouhal number, Reynolds numbers and inlet conditions.

REFERENCES


